Connecticut Intermediate Algebra Student Workbook

Version 1.3
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About this Workbook

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Mathematics instructors at Scottsdale Community College in Scottsdale, Arizona originally created this workbook. Faculty from Housatonic Community College and Middlesex Community College collaborated and edited the book to fit Connecticut's Intermediate Algebra outcomes. The included content is designed to lead students through Intermediate Algebra, from a functions modeling approach, and to develop a deep understanding of the concepts associated with functions, data and change. The included curriculum is broken into eleven lessons. Each lesson includes the following components:

MINI-LESSON

- The **Mini-Lesson** is the main instructional component for each lesson.
- Ideas are introduced with practical applications.
- **Worked Examples** are provided for each topic in the Mini-Lesson. Read through these examples carefully. Use these as a guide for completing similar problems.
- **Media Examples** can be worked by watching online videos and taking notes/writing down the problem as written by the instructor. Video links can be found within the MyOpenMath (MOM) Online Homework Assessment System.
- **You-Try** problems help reinforce Lesson concepts and should be worked in the order they appear showing as much work as possible.

PRACTICE PROBLEMS

- This section follows the Mini-Lesson. If you are working through this material on your own, the recommendation is to work all practice problems. If you are using this material as part of a formal class, your instructor will provide guidance on which problems to complete. Your instructor will also provide information on accessing answers/solutions for these problems.

LESSON ASSESSMENT

- The last part of each Lesson is a short assessment. If you are working through this material on your own, use these assessments to test your understanding of the lesson concepts. Take the assessments without the use of the book or your notes and then check your answers. If you are using this material as part of a formal class, your instructor will provide instructions for completing these problems and for obtaining solutions to the practice problems.

MYOPENMATH ONLINE HOMEWORK ASSESSMENT SYSTEM

- If you are using these materials as part of a formal class and your class utilizes an online homework/assessment system, your instructor will provide information as to how to access and use that system in conjunction with this workbook.

OVERVIEW OF TOPICS

- Lessons 1-4: Linear Functions
- Lessons 5-6: Quadratic Functions
- Lesson 7: Radical Functions
- Lesson 8: Rational Functions
- Lessons 9-10: Exponential Functions
- Lesson 11: Review

Table of Contents

Lesson 1 – Introduction to Functions

Throughout this class, we will be looking at various Algebraic Functions and the characteristics of each. Before we begin, we need to review the concept of what a Function is and look at the rules that a Function must follow. We also need to investigate the different ways that we can represent a Function. It is important that we go beyond simple manipulation and evaluation of these Functions by examining their characteristics and analyzing their behavior. Looking at the Functions modeled as Graphs, Tables and Sets of Ordered Pairs is critical to accomplishing that goal.

Lesson Topics:

Section 1.1 What is a function?

- Definition of a function
- Independent and Dependent Variables

Section 1.2 Multiple Representations of Functions

- Sets of ordered pairs (input, output)
- Tables
- Graphs
- Vertical Line Test
- Behavior of Graphs

Section 1.3 Function Notation

- Function evaluation
- Working with input and output
- Multiple Representations
- Graphs and tables with a graphing calculator

Section 1.4 Domain and Range

- Definitions
- Multiple Representations
- Restricting Domain and Range (calculator)

Section 1.5 Applications of Functions

- Criteria for a good graph
- Practical Domain and Range

Mini-Lesson 1

Section 1.1 – What is a Function?

Intermediate Algebra is a study of functions and their characteristics. In this class, we will study LINEAR, EXPONENTIAL, QUADRATIC, RATIONAL, & RADICAL functions. Before we learn the specifics of these functions, we need to review/learn the language and notation of FUNCTIONS.

What is a Function?

The concept of "function" is one that is very important in mathematics. The use of this term is very specific and describes a particular relationship between two quantities: an input quantity and an output quantity. Specifically, a relationship between two quantities can be defined as a function if it is the case that, "each input value is associated with only one output value."

Why Do We Care About Functions?

Imagine that you are a nurse working the emergency room of a hospital. A very sick person arrives. You know just the medicine needed but you are unsure the exact dose. First, you determine the patient's weight (200 pounds). Then you look at the table below and you will see the given dosage information:

Weight in lbs.	mL of Medicine
200	10
200	100

You are immediately confused and very concerned. How much medicine do you give? 10 ml or 100 ml? One amount could be too much and the other not enough. How do you choose the correct amount? What you have here is a situation that does NOT define a function (and would not occur in real life). In this case, for the input value 200 lbs, there are two choices for the output value. If you have a function, you will not have to choose between output values for a given input. In the real case of patients and medicine, the dosage charts are based upon functions.

A More Formal Definition of Function:

A FUNCTION is a rule that assigns each input value to a single, unique, output value.

Problem 1	**MEDIA EXAMPLE – Do The Data Represent A Function?**

The table below gives the height H, in feet, of a golf ball t seconds after being hit.

t = Time (in seconds)	H = Height (in feet)
0	0
1	80
2	128
3	144
4	128
5	80
6	0

a) Identify the input quantity (include units).____

Identify the input variable.____

Identify the output quantity (include units)._______________________________________

Identify the output variable.____

b) Write the data as a set of ordered pairs.

c) Interpret the meaning of the ordered pair (3, 144).

d) Is height of the golf ball a function of time? Why or why not?

e) Is time a function of the height of the golf ball? Why or why not?

Problem 2	**WORKED EXAMPLE – Investigating Functional Relationships**

Let's investigate the functional relationship between the two quantities, "numerical grade" and "letter grade". First, let Numerical Grade be the input quantity and Letter Grade be the output quantity. Below is a sample data set that is representative of the situation.

Numerical grade	Letter Grade
95	A
92	A
85	B
73	C

The numbers above are made up to work with this situation. Other numbers could be used. We are assuming a standard 90, 80, 70, etc… grading scale. Hopefully you can see from this data that no matter what numerical value we have for input, there is only one resulting letter grade. Notice that the repeated outputs "A" are not a problem since the inputs are different. You can uniquely predict the output for any numerical grade input.

So, from this information we can say that Letter Grade (output) is a function of Numerical Grade (input).

Now let's switch the data set above.

Letter Grade	Numerical Grade
A	95
A	92
B	85
C	73

Can you see there is a problem here? If you say that you have an A in a class, can you predict your numerical grade uniquely? No. There are a whole host of numerical scores that could come from having an A. The same is true for all the other letter grades as well. Therefore, Numerical Grade (output) is NOT a function of Letter Grade (input).

Summary:

- Letter Grade IS a function of Numerical Grade but
- Numerical Grade is NOT a function of Letter Grade

Additional Terminology

In the language of functions, the phrase INDEPENDENT VARIABLE means input and the phrase DEPENDENT VARIABLE means output. The dependent variable (output) "depends on" or is a "function of" the independent variable (input).

Problem 3	**YOU TRY – Do The Data Represent A Function?**

The table below gives the value of a car *n* years after purchase

n = Time (in years)	V = Value (in dollars)
0	32540
1	28310
2	24630
3	21428
4	18642
5	16219
6	14110

a) Identify the input quantity (include units).____

Identify the output quantity (include units).____

b) Identify the dependent variable.____

Identify the independent variable.____

c) Interpret the meaning of the ordered pair (2, 24630).

d) Is the value of the car a function of time? Why or why not?

Section 1.2 – Multiple Representations of Functions

Problem 4	**MEDIA EXAMPLE – Determine Functional Relationships Using Multiple Representations**

SETS OF ORDERED PAIRS (input, output)
Which of the following represent functional relationships?

{(-3, 2), (5, 0), (4, -7)}

{(0, 2), (5, 1), (5, 4)}

{(-3, 2), (5, 2), (4, 2)}

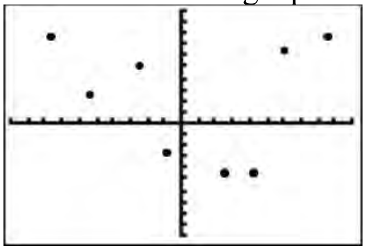
TABLES
Which of the following represent functional relationships?

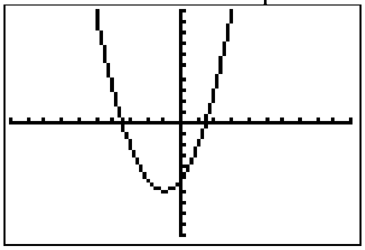
x	*y*
2	52
4	41
4	30
7	19

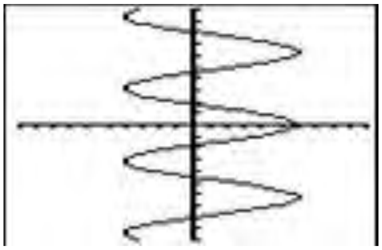
x	*y*
3	128
11	64
24	
38	16

x	*y*
0	4
1	4
2	4
3	4

GRAPHS
Which of the following represent functional relationships?





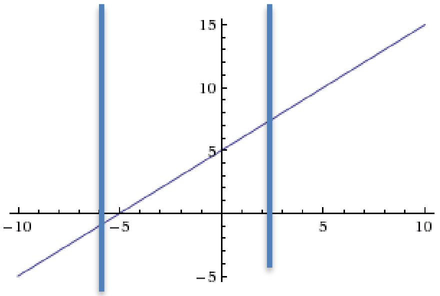


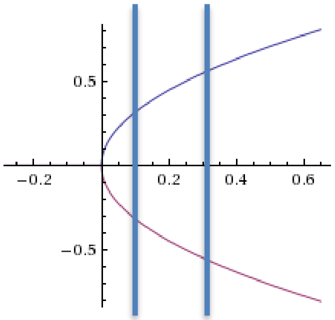
THE VERTICAL LINE TEST

- If all vertical lines intersect the graph of a relation at not more than one point, the relation *is* also a function. One and only one output value exists for each input value.
- If any vertical line intersects the graph of a relation at more than one point, the relation "fails" the test and is NOT a function. More than one output value exists for some (or all) input value(s).

Problem 5	**WORKED EXAMPLE – Determine Functional Relationships Using Multiple Representations**

The table below shows 3 different representations for two relationships. Determine which relationship defines a function.

<table>
<tr><th></th><th>Functions</th><th>Not Functions</th></tr>
<tr><td>Set of Ordered Pairs</td><td>{(-7, 6), (-3, 3), (1, 8), (5, 8), (11, 0)}
No input value is repeated in an ordered pair.</td><td>{(8, 3), (6, 1), (8, -1), (6, 11), (2, -5)}
Two of the listed input values (6 & 8) are associated with more than one output value.</td></tr>
<tr><td>Table</td><td>
<table>
<tr><td><i>x</i></td><td>-4</td><td>-2</td><td>0</td><td>1</td><td>5</td></tr>
<tr><td><i>y</i></td><td>8</td><td>3</td><td>5</td><td>3</td><td>10</td></tr>
</table>
OR
<table>
<tr><th><i>x</i></th><th><i>y</i></th></tr>
<tr><td>-4</td><td>8</td></tr>
<tr><td>-2</td><td>3</td></tr>
<tr><td>0</td><td>5</td></tr>
<tr><td>1</td><td>3</td></tr>
<tr><td>5</td><td>10</td></tr>
</table>
All input values are associated with one, unique output value.</td><td>
<table>
<tr><td><i>x</i></td><td>0</td><td>1</td><td>1</td><td>4</td><td>4</td></tr>
<tr><td><i>y</i></td><td>0</td><td>1</td><td>-1</td><td>2</td><td>-2</td></tr>
</table>
OR
<table>
<tr><th><i>x</i></th><th><i>y</i></th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td>1</td><td>-1</td></tr>
<tr><td>4</td><td>2</td></tr>
<tr><td>4</td><td>-2</td></tr>
</table>
Two of the listed input values (1 & 4) are associated with more than one output value.</td></tr>
<tr><td>Graph</td><td>

No vertical line intersects the graph in more than one point.
We say the graph PASSES the VERTICAL LINE TEST.</td><td>

Vertical lines intersect the graph at more than one point meaning inputs are repeated with different outputs.
We say that the graph FAILS the VERTICAL LINE TEST.</td></tr>
</table>

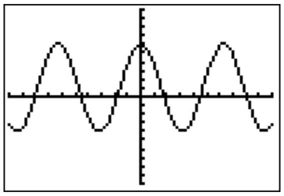
Problem 6	**YOU TRY – Determine Functional Relationships Using Multiple Representations**

Which of the following represent functional relationships?

A

{(4, 1), (7, 1), (-3, 1), (5, 1)}

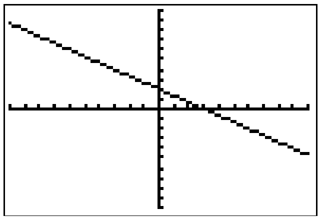
B



C

x	y
5	4
5	6
5	8
5	1

D



E

{(3, 5), (3, 6), (8, 1), (5, 4)}

F

x	y
0	2
3	2
5	3
11	5

Problem 7	**MEDIA EXAMPLE – Does the Statement Describe A Function?**

Explain your choice for each of the following. Remember when the word "function" is used, it is in a purely MATHEMATICAL sense, not in an everyday sense.

a) Is the number of children a function of parents' income?

b) Is your weekly pay a function of the number of hours you work each week? (Assume you work at an hourly rate job with no tips).

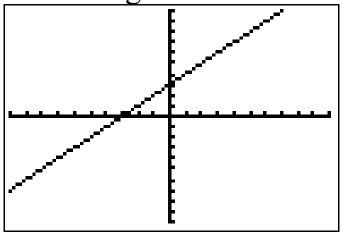
Problem 8	**WORKED EXAMPLE – Behavior of Functions**

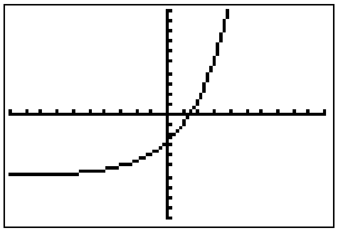
A function is:

- INCREASING if the outputs get larger,
- DECREASING if the outputs get smaller,
- CONSTANT if the outputs do not change.

NOTE: We read graphs just like we read a book…from left to right.

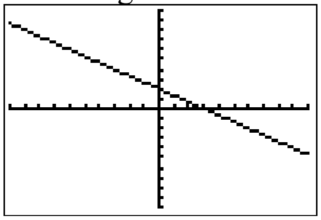
a) The following functions are INCREASING

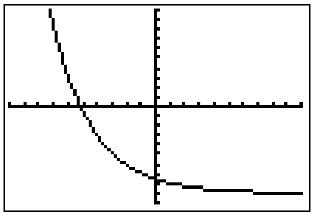




x	y
0	4
1	6
2	12
3	24

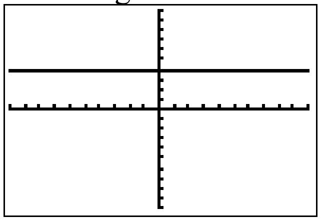
b) The following functions are DECREASING

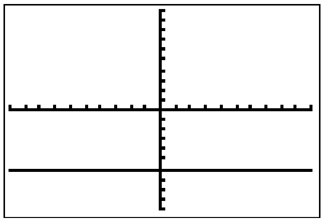




x	y
0	10
1	5
2	0
3	-5

c) The following functions are CONSTANT





x	y
0	4
1	4
2	4
3	4

Section 1.3 – Function Notation

FUNCTION NOTATION is used to indicate a functional relationship between two quantities as follows:

Function Name (INPUT) = OUTPUT

So, the statement $f(x) = y$ would refer to the function f, and correspond to the ordered pair (x,y), where x is the input variable, and y is the output variable.

Function Evaluation: To evaluate a function at a particular value of the input variable, replace each occurrence of the input variable with the given value and compute the result.

Note: Use of ( ) around your input value, especially if the input is negative, can help achieve correct results.

Problem 9	**MEDIA EXAMPLE – Function Evaluation**

Given $f(x) = 2x - 5$, evaluate $f(2)$, $f(-1)$, $f(x+1)$ and $f(-x)$.

Problem 10	**WORKED EXAMPLE – Function Evaluation**

If $f(x)=5x^2-3x-10$, find a) $f(2)$, b) $f(-1)$ c) $f\left(\frac{-1}{2}\right)$.

a) $f(2)=5(2)^2-3(2)-10=5(4)-6-10=20-16=4$

b) $f(-1)=5(-1)^2-3(-1)-10=5(1)+3-10=5+3-10=-2$

c) $f\left(-\frac{1}{2}\right)=5\left(-\frac{1}{2}\right)^2-3\left(-\frac{1}{2}\right)-10=5\left(\frac{1}{4}\right)+\frac{3}{2}-10=\frac{5}{4}+\frac{3}{2}-10=-\frac{29}{4}$

When working with FUNCTIONS, there are two main questions we will ask and solve as follows:

- Given a particular INPUT value, what is the corresponding OUTPUT value?
- Given a particular OUTPUT value, what is the corresponding INPUT value?

Problem 11	**MEDIA EXAMPLE – Working with Input and Output**

Given $f(x)=2x+5$, determine each of the following. Write your answers as ordered pairs.

GIVEN INPUT FIND OUTPUT

Find $f(0)$

Find $f(-2)$

GIVEN OUTPUT FIND INPUT

Find x if $f(x)=7$

Find x if $f(x)=-11$

Problem 12	**YOU TRY – Working with Input and Output**

Given $f(x) = -3x - 4$, compute each of the following. Show all steps, and write your answers as ordered pairs. Write answers as integers or reduced fractions (no decimals).

a) Find $f(2)$

b) Find x if $f(x) = 7$

c) Find $f(-3)$

d) Find x if $f(x) = 10$

e) Find $f(-x)$

f) Find $f(x-5)$

g) Find $f\left(-\frac{1}{2}\right)$

Problem 13	**MEDIA EXAMPLE – Working with Function Notation Using a Set of Ordered Pairs**

The function $g(x)$ is shown below

$$g = \{(1,3),(5,2),(8,3),(6,-5)\}$$

$g(1) =$ __________ Find x if $g(x) = -5$. $x =$ __________

Find x if $g(x) = 3$. $x =$ ____________________

Problem 14	**MEDIA EXAMPLE – Working with Function Notation Using a Table**

The function $V(n)$ is shown below gives the value, V, of an investment (in thousands of dollars) after n months.

n	1	2	3	4
$V(n)$	2.31	3.02	5.23	3.86

Identify the input quantity (include units).________________________________

Identify the output quantity (include units).________________________________

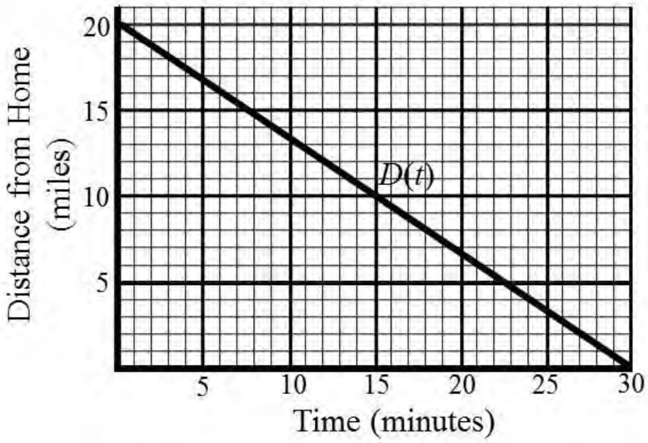
Write a sentence explaining the meaning of the statement $V(1) = 2.31$.

Determine $V(3)$ and write a sentence explaining its meaning.

For what value of n is $V(n) = 3.02$? Interpret your answer in a complete sentence.

Problem 15	**MEDIA EXAMPLE – Working with Function Notation Using a Graph**

The function $D(t)$ below shows a person's distance from home as a function of time.



Identify the input quantity (include units).________________________________

Identify the output quantity (include units).________________________________

Write a sentence explaining the meaning of the statement $D(15) = 10$.

Determine $D(0)$ and write a sentence explaining its meaning.

For what value of t is $D(t) = 0$? Interpret your answer by writing a complete sentence.

Problem 16	**MEDIA EXAMPLE – Using Your Graphing Calculator to create a table and Graph of a Function**

Consider the function $y = 5 - 2x$

a) Use your graphing calculator to complete the table below

x	0	3	7	9	12
y					

b) Use your graphing calculator to sketch the graph of $y = 5 - 2x$.

Use the standard viewing window (ZOOM→6) xmin= -10, xmax=10, ymin=-10, ymax=10 ,

Draw what you see on your calculator screen.

c) Use your graphing calculator to sketch the graph of $y = 5 - 2x$.

Use viewing window xmin= 0, xmax= 3, ymin= 0, ymax= 5 ,

Draw what you see on your calculator screen.

Section 1.4 – Domain and Range

The DOMAIN of a function is the set of all possible values for the input quantity.

The RANGE of a function is the set of all possible values for the output quantity

Problem 17	**MEDIA EXAMPLE – Domain and Range, Multiple Representations**

SET OF ORDERED PAIRS
Determine the domain and range of the function $P(x) = \{(2, 3),\ (4, -5),\ (6, 0),\ (8, 5)\}$

Domain: ______________________________

Range: ______________________________

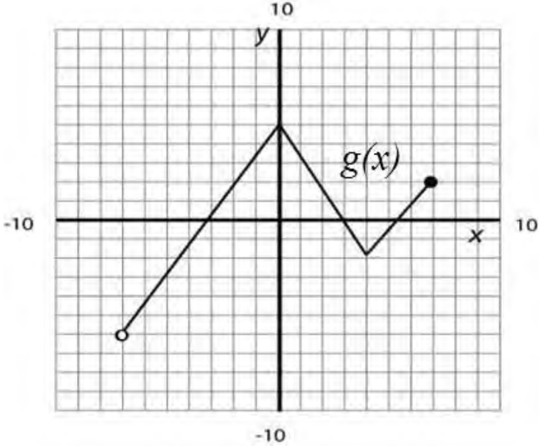
TABLE
Determine the domain and range of the function R(t) defined below.

t	0	2	5	8	11
R(t)	23	54	66	87	108

Domain: ______________________________

Range: ______________________________

GRAPH
Determine the domain and range of the function $g(x)$ defined below.



Domain of $g(x)$:

Inequality Notation Interval Notation

Range of $g(x)$:

Inequality Notation Interval Notation

Problem 18	**MEDIA EXAMPLE – Restricting the Domain and Range (Calculator)**

Graph the following function on your graphing calculator restricting the input window to Xmin = -5 and Xmax = 5 and draw an accurate sketch here [Go to Y= and type in the equation. Then go to Window and enter -5 for Xmin and 5 for Xmax. Leave Ymin at -10 and Ymax at 10]. Indicate the domain and range given the window above.

a) $y = x - 1$

Domain:______________________________

Range:_______________________________

b) If the input and output are not restricted as above, indicate the domain and range for this function.

Domain:

Range:

Problem 19	**YOU TRY – Domain and Range, Multiple Representations**

Find the domain and range for the functions below. Use proper notation for your domain/range responses.

a) Set of ordered pairs

$D(r) = \{(7,8),(8,12),(11,21)\}$

Domain: ______________________________

Range: ______________________________

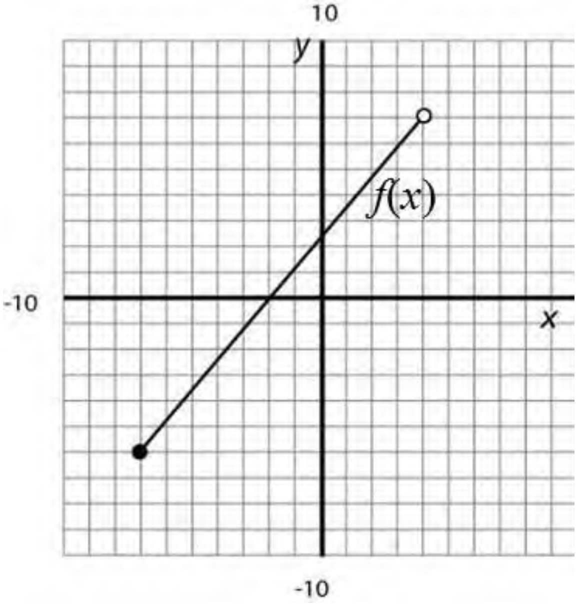
b) Table of values

n	A(n)
3	51
6	42
8	33

Domain: ______________________________

Range: ______________________________

c) Graph



Domain of $f(x)$

Inequality Notation Interval Notation

Range of $f(x)$

Inequality Notation Interval Notation

Section 1.5 – Applications of Functions

Criteria for a GOOD GRAPH:

1. The horizontal axis should be properly labeled with the name and units of the input quantity.

2. The vertical axis should be properly labeled with the name and units of the output quantity.

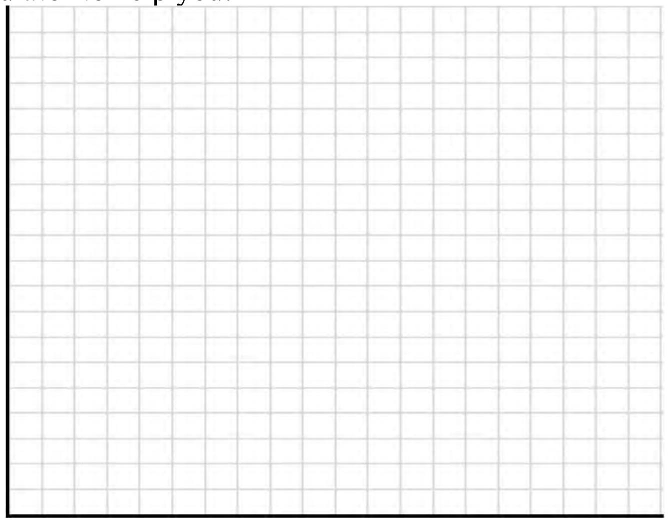
3. Use an appropriate scale.
   - Start at or just below the lowest value.
   - End at or just above the highest value.
   - Scale the graph so the adjacent tick marks are equal distance apart.
   - Use numbers that make sense for the given data set.
   - The axes meet at (0,0) Use a "//" between the origin and the first tick mark if the scale does not begin at 0.

4. All points should be plotted correctly, and the graph should be neat and uncluttered.

Problem 20	**MEDIA EXAMPLE – Understanding Applications of Functions**

Suppose that the cost to fill your 15-gallon gas tank is determined by the function C(g) = 3.29g where C is the output (cost in $) and g is the input (gallons of gas).

a) Draw a GOOD graph of this function in the space below. Provide labels for your axes. You may use the graphing feature of your calculator to help you.



b) Use the Table feature of your graph and identify the first and last ordered pairs that are on the graph (based on the information above). [2nd>Graph will take you to the table]. Include both ordered pairs and function notation.

c) What is the INPUT quantity (including units) for this function? Name the smallest and largest possible input quantity then use this information to identify the PRACTICAL DOMAIN.

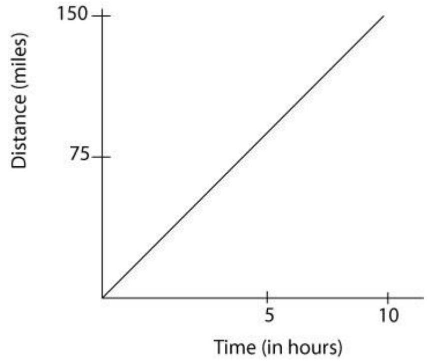
d) What is the OUTPUT quantity (including units) for this function? Name the smallest and largest possible output quantity then use this information to identify the PRACTICAL RANGE.

Practical Domain: The PRACTICAL DOMAIN of a function is the set of all possible input values that are realistic for a given problem.

Practical Range: The PRACTICAL RANGE of a function is the set of all possible output values that are realistic for a given problem.

Problem 21 WORKED EXAMPLE – Practical Domain and Range

Let the function $M(t) = 15t$ represent the distance you would travel bicycling t hours. Assume you can bike no more than 10 hours. Find the practical domain and practical range for this function.

BEGIN by drawing an accurate graph of the situation. Determine the smallest and largest input values of the function then do the same thing for the output values.	
PRACTICAL DOMAIN In this situation, the input values you can use are related to biking and the input is TIME. You are told you can bike no more than 10 hours. You also cannot bike a negative number of hours but you CAN bike 0 hours. Therefore, the Practical Domain is $0 \le t \le 10$ hours This means “all the values of t between and including 0 and 10”.	PRACTICAL RANGE In this situation, the outputs represent distances traveled depending on how long you bike. Looking at the endpoints for Practical Domain, you can find you Practical Range as follows: $M(0) \le M(t) \le M(10)$ Thus, $0 \le M(t) \le 150$ miles is your Practical Range This means you can bike a minimum of 0 miles and a maximum of 150 miles in this situation.

Problem 22	**YOU TRY – Applications of Functions**

A local towing company charges $3.25 per mile driven plus a base fee of $30.00. They tow a maximum of 25 miles.

a) Let C represent the total cost of any tow and x represent miles driven. Using correct and formal function notation, write a function that represents total cost as a function of miles driven.

b) Identify the practical domain of this function by filling in the blanks below.

Minimum miles towed $\leq x \leq$ Maximum miles towed

Practical Domain: ______________ $\leq x \leq$ _______________

c) Identify the practical range of this function by filling in the blanks below.

Minimum Cost $\leq C(x) \leq$ Maximum Cost

Practical Range: ______________ $\leq C(x) \leq$ _______________

d) Write a complete sentence to explain the meaning of $C(60)=225$ in words.

e) Use your function from part a) to find $C(15)$. Write your answer as ordered pair then explain its meaning in a complete sentence.

f) Use your function from part a) to determine the value of x when $C(x)=30$. Write your answer as ordered pair then explain its meaning in a complete sentence.

Problem 23 YOU TRY – Applications of Functions

The value V (in dollars) of a washer/dryer set decreases as a function of time *t* (in years). The function $V(t) = -100t + 1200$ models this situation. You own the washer/dryer set for 12 years.

a) Identify the input quantity (including units) and the input variable.

b) Identify the output quantity (including units) and the output variable.

c) Fill in the table below.

t	0	6	12
V(*t*)			

d) Draw a GOOD graph of this function in the space below. Provide labels for your axes. Plot and label the ordered pairs from part c). You may use the graphing feature of your calculator to help you.

e) A washer/dryer set that is worth $400 would be how old?
Hint: This is a GIVEN OUTPUT FIND INPUT question. You must show work.

f) After 2 years, how much would the washer/dryer set be worth?
Hint: This is a GIVEN INPUT FIND OUTPUT question. You must show work.

g) What is the practical domain for V(*t*)?

Inequality notation: ____

Interval notation: ____

h) What is the practical range for V(*t*)?

Inequality notation: ____

Interval notation: ____

Lesson 1 Practice Problems

Round to two decimal places unless stated otherwise.

Section 1.1: What is a Function?

1. The table below gives the distance D, in kilometers, of a GPS satellite from Earth t minutes after being launched.

t = Time (in minutes)	D = Distance (in km)
0	0
20	4003
40	9452
60	14,232
80	18,700
100	20,200
120	20,200

a) Identify the input quantity (include units).______________________________________

Identify the input variable.____

Identify the output quantity (include units).______________________________________

Identify the output variable.____

b) Write the data as a set of ordered pairs.

c) Interpret the meaning of the ordered pair (40, 9452).

d) Is distance of the satellite a function of time? Why or why not?

e) Is time a function of the distance of the satellite from Earth? Why or why not?

2. The table below gives the number of Gene copies, G, t minutes after observation.

t = Time (in minutes)	G = number of Gene Copies
0	52
3	104
5	165
6	208
8	330
10	524
12	832

a) Identify the input quantity (include units).______________________________

Identify the input variable.______________________________

Identify the output quantity (include units).______________________________

Identify the output variable.______________________________

b) Write the data as a set of ordered pairs.

c) Interpret the meaning of the ordered pair (6, 208).

d) Is the number of Gene copies a function of time? Why or why not?

e) Is time a function of the number of Gene copies? Why or why not?

3. The table below gives the number of homework problems, H, that Tara has completed t minutes after she began her homework.

t = Time (in minutes)	H = number of homework problems completed
0	0
10	3
20	8
30	8
40	15
50	17
60	20

a) Identify the input quantity (include units).____

Identify the input variable.____

Identify the output quantity (include units).______________________________________

Identify the output variable.____

b) Write the data as a set of ordered pairs.

c) Interpret the meaning of the ordered pair (40, 15).

d) Is the number of homework problems completed a function of time? Why or why not?

e) Is time a function of the number of homework problems completed? Why or why not?

4. The table below gives the number of hot dogs, H, that a competitive hot dog eater has eaten t minutes after the start of the competition.

t = Time (in minutes)	H = number of hotdogs eaten
0	0
1	8
3	23
5	37
7	50
9	63
10	68

a) Identify the input quantity (include units)._______________________________________

Identify the input variable.____

Identify the output quantity (include units).______________________________________

Identify the output variable.____

b) Write the data as a set of ordered pairs.

c) Interpret the meaning of the ordered pair (7, 50).

d) Is the number of hot dogs eaten a function of time? Why or why not?

e) Is time a function of the number of hot dogs eaten? Why or why not?

Section 1.2: Multiple Representations of Functions

5. Determine whether the following sets of ordered pairs represent a functional relationship. Justify your answer.

   a) $R = \{(2, 4), (3, 8), (-2, 6)\}$

   b) $T = \{(3, -2), (4, -1), (5, 8), (3, -2)\}$

   c) $L = \{(3, -5), (1, -2), (2, -2), (3, 5)\}$

   d) $A = \{(5, -5), (6, -5), (7, -5\}$

   e) $F = \{(2, -3), (6, \quad), (4, 8)\}$

6. Determine whether the following tables of values represent a functional relationship. Justify your answer.

a)

x	$f(x)$
3	0
1	5
2	8
3	12
4	14

b)

x	$g(x)$
0	0
1	1
2	–1
3	2
4	–2

c)

x	$f(x)$
0	–3
1	–4
2	–5
3	
4	–6

d)

r	−1	1	2	1
$h(r)$	3	5	3	5

e)

s	0	1	2	3	4
$l(s)$	5	10	15		25

f)

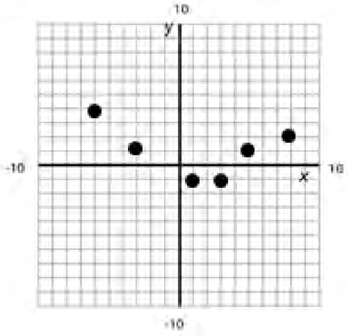
t	0	1	2
$p(t)$	43	45	43

g)

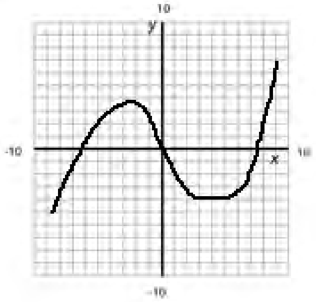
s	−1	−1	−1	−1
$t(s)$	8	12	−4	−9

7. Determine whether the following graphs represent a functional relationship. Justify your answer.

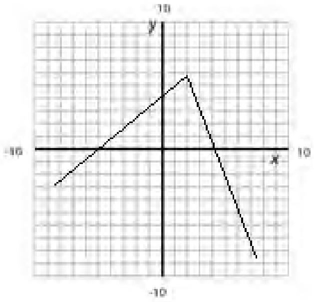
a)



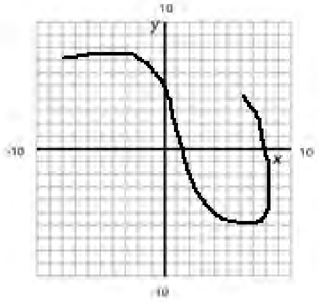
b)



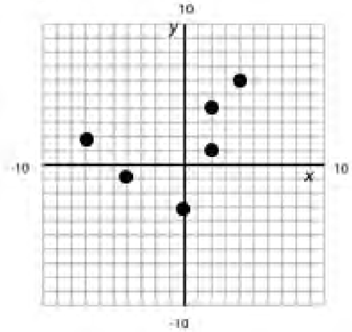
c)



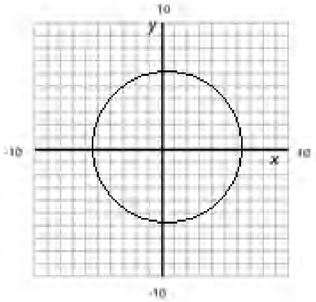
d)



e)



f)



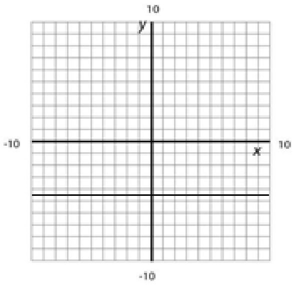
8) Determine whether the following scenarios represent functions. Explain your choice for each of the following. Remember when the word "function" is used, it is in a purely MATHEMATICAL sense, not in an everyday sense.

a) Is a person's height a function of their age?

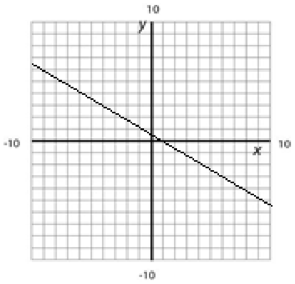
b) Is a person's age a function of their date of birth?

9. Determine whether the following functions represented by the graphs below are increasing, decreasing, or constant.

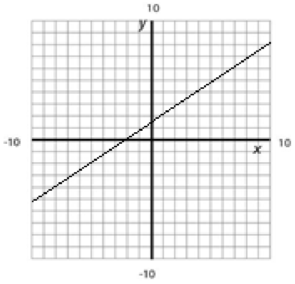
a.



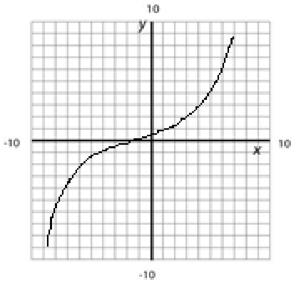
b.



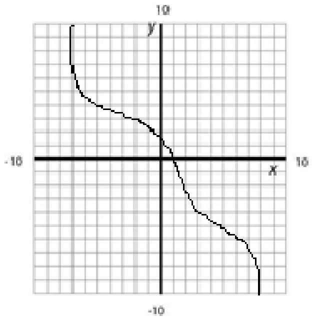
c.



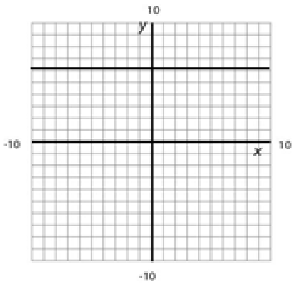
d.



e.



f.



10. Determine whether the following functions represented by the tables below are increasing, decreasing, or constant.

a)

x	$f(x)$
-2	5
-1	8
0	23
1	37
2	49

b)

t	$s(t)$
4	5
7	3
12	-8
13	-12
17	-25

c)

x	$g(x)$
0	5
1	5
2	5
3	5
4	5

d)

x	0	1	2	3
$h(x)$	2	2	2	2

e)

x	-5	1	8	9
$h(x)$	27	26	24	23

f)

x	0	1	2	3
$h(x)$	-22	-20	-18	-15

Section 1.3: Function Evaluation

11. Given the function $f(x) = -x + 6$, evaluate each of the following

a) $f(2) =$

b) $f(-1) =$

c) $f(0) =$

12. Given the function $s(t) = 14 - 2t$, evaluate each of the following:

    a) $s(-3) =$

    b) $s(4) =$

    c) $s(0) =$

13. Given the function $h(c) = 2c^2 - 3c + 4$, evaluate each of the following:

    a) $h(-2) =$

    b) $h(3) =$

    c) $h(0) =$

14. Given the function $g(x) = -x^2 + 3x$, evaluate each of the following:

    a) $g(-3) =$

    b) $g(4) =$

    c) $g(0) =$

15. Given the function $f(x) = -x + 6$, evaluate each of the following

    a) $f(2x) =$

    b) $f(x-3) =$

16. Given the function $s(t) = 14 - 2t$, evaluate each of the following:

    a) $s(3t) =$

    b) $s\left(\frac{1}{4}t\right) =$

    c) $s(t+4) =$

17. Given the function $h(c)=c^2+1$, evaluate each of the following:

a) $h(-2c)=$

b) $h(c-1)=$

c) $h(x+2)=$

18. Given $f(x) = 3x - 6$, determine each of the following. Also determine if you are given an input or output and whether you are finding an input or output and write your result as an ordered pair.

a) Find $f(2)$ =

Given input or output?______________

Finding input or output?_____________

Ordered pair:_______________________

b) Find x if $f(x)$= 3

Given input or output?______________

Finding input or output?_____________

Ordered pair:_______________________

c) Find $f(-4)$ =

Given input or output?______________

Finding input or output?_____________

Ordered pair:_______________________

d) Find x if $f(x) = -12$

Given input or output?______________

Finding input or output?_____________

Ordered pair:_______________________

19. Given $g(x)=\frac{3}{2}x-\frac{1}{2}$, determine each of the following. Write your final result as a fraction when appropriate.

a) Find $g(4)$ =

Given input or output?______________

Finding input or output?_____________

Ordered pair:______________________

b) Find x if $g(x)= 3$

Given input or output?______________

Finding input or output?_____________

Ordered pair:______________________

c) Find $g(-8)$ =

Given input or output?______________

Finding input or output?_____________

Ordered pair:_____________________

d) Find x if $g(x) = -\frac{7}{2}$

Given input or output?______________

Finding input or output?_____________

Ordered pair:______________________

20. Use the table below to find the function values.

t	−8	−3	2	7	12
k(t)	14	7	0	−7	−14

a) $k(7) =$

$b)$ $k(-3) =$

$c)$ $k(-8) =$

21. Given the table for the function below, determine each of the following. Also determine if you are given an input or output and whether you are finding an input or output and write your result as an ordered pair.

x	−6	−4	−2	0	2
f(*x*)	9	3	−5	−12	−17

a) Find *x* if $f(x) = -12$

Given input or output?______________

Finding input or output?_____________

Ordered pair:______________________

b) Find $f(-4)$ =

Given input or output?______________

Finding input or output?_____________

Ordered pair:______________________

c) Find *x* if $f(x)$= 3

Given input or output?______________

Finding input or output?_____________

Ordered pair:______________________

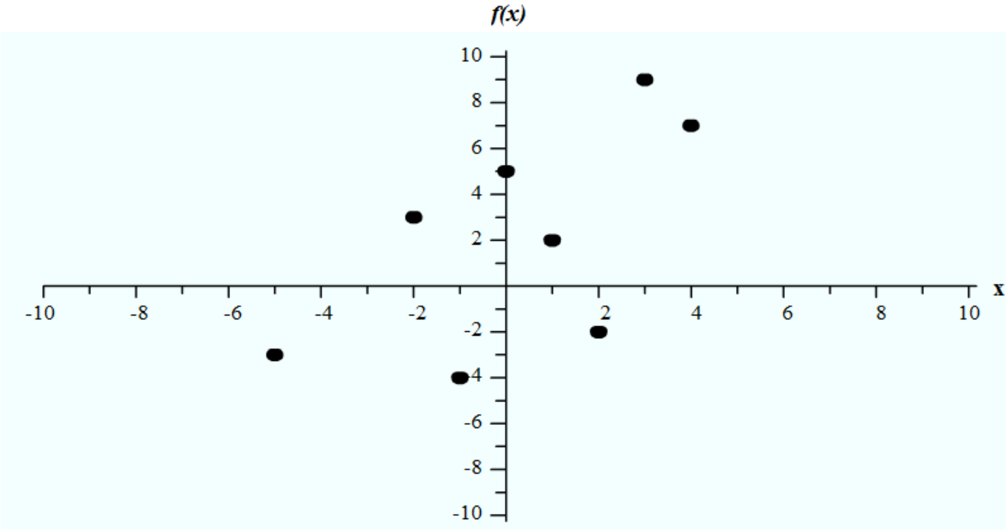
d) Find $f(2)$ =

Given input or output?______________

Finding input or output?_____________

Ordered pair:______________________

22. Given the graph for the function below, determine each of the following. Also determine if you are given an input or output and whether you are finding an input or output and write your result as an ordered pair.



a) Find x if $f(x) = 5$

Given input or output?______________

Finding input or output?_____________

Ordered pair:______________________

b) Find $f(-2)$ =

Given input or output?______________

Finding input or output?_____________

Ordered pair:______________________

c) Find x if $f(x)$= 3

Given input or output?______________

Finding input or output?_____________

Ordered pair:______________________

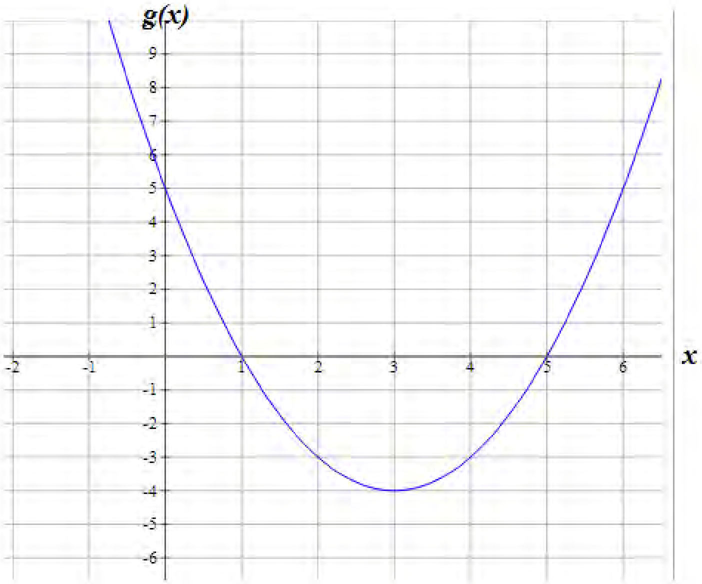
d) Find $f(3)$ =

Given input or output?______________

Finding input or output?_____________

Ordered pair:______________________

23. Given the graph for the function below, determine each of the following. Also determine if you are given an input or output and whether you are finding an input or output and write your result as an ordered pair.



a) Find any *x-values* where $g(x) = 5$

Given input or output?_______________

Finding input or output?______________

Ordered pair:________________________

b) Find $g(2)$ =

Given input or output?_______________

Finding input or output?______________

Ordered pair:________________________

c. Find any *x-values* where $g(x) = 0$

Given input or output?_______________

Finding input or output?______________

Ordered pair:________________________

d) Find $g(3)$ =

Given input or output?_______________

Finding input or output?______________

Ordered pair:________________________

24. Consider the function $y = 2x - 3$

a) Use your graphing calculator to complete the table below

x	-3	-1	0	1	3
y					

b) Use your graphing calculator to sketch the graph of $y = 2x - 3$

Use the standard viewing window (ZOOM→6) xmin= -10, xmax=10, ymin=-10, ymax=10 ,

Draw what you see on your calculator screen.

c) Use your graphing calculator to sketch the graph of $y = 2x - 3$.

Use viewing window xmin= 0, xmax= 3, ymin= 0, ymax= 5 ,

Draw what you see on your calculator screen.

25. Consider the function $f(x) = -3x + 4$

a) Use your graphing calculator to complete the table below

x	-3	-1	0	1	3
$y=f(x)$					

b) Use your graphing calculator to sketch the graph of $f(x) = -3x + 4$

Use the standard viewing window (ZOOM→6) xmin= -10, xmax=10, ymin=-10, ymax=10 ,

Draw what you see on your calculator screen.

c) Use your graphing calculator to sketch the graph of $f(x) = -3x + 4$

Use viewing window xmin= 0, xmax= 5, ymin= −15, ymax= 5 ,

Draw what you see on your calculator screen.

Section 1.4: Domain and Range

26. For each set of ordered pairs, determine the domain and the range.

a) $g = \{(3, -2), (5, -1), (7, 8), (9, -2), (11, 4), (13, -2)\}$

Domain:

Range:

b) $f = \{(-2, -5), (-1, -5), (0, -5), (1, -5)\}$

Domain:

Range:

c) $h = \{(-3, 2), (1, -5), (0, -3), (4, -2)\}$

Domain:

Range:

27. For each table of values, determine the domain and range of the function.

a)

x	$f(x)$
–10	3
–5	8
0	12
5	15
10	18

Domain:

Range:

b)

x	$g(x)$
–20	–4
–10	14
0	32
10	50
20	68
30	86

Domain:

Range:

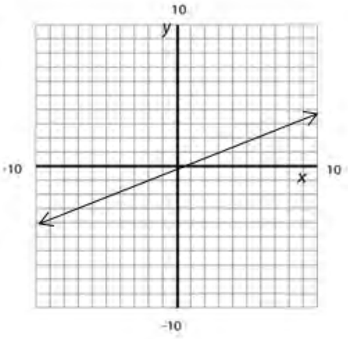
c)

h	8	9	10	11	12	1	2	3	4
$T(h)$	54	62	66	69	72	73	74	73	72

Domain: Range:

28. For each graph, determine the domain and range of the function. Use inequality and interval notation when appropriate.

a)



Domain:

Inequality notation:__________________

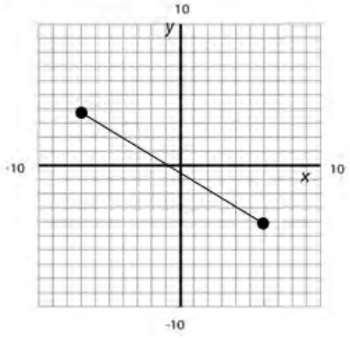
Interval notation:____________________

Range:

Inequality notation:__________________

Interval notation:

b)



Domain:

Inequality notation:__________________

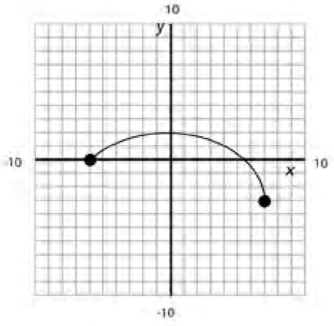
Interval notation:____________________

Range:

Inequality notation:__________________

Interval notation: ________

c)



Domain:

Inequality notation:__________________

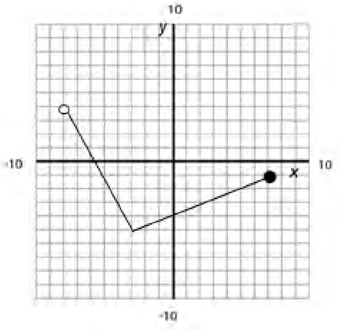
Interval notation:____________________

Range:

Inequality notation:__________________

Interval notation:____________________

d)



Domain:

Inequality notation:__________________

Interval notation:____________________

Range:

Inequality notation:__________________

Interval notation:____________________

Section 1.5: Applications of Functions

29. A local window washing company charges \$0.50 per window plus a base fee of \$20.00 per appointment. They can wash a maximum of 200 windows per appointment.

a) Let C represent the total cost of an appointment and w represent the number of windows washed. Using correct and formal function notation, write a function that represents total cost as a function of windows washed.

b) Identify the practical domain of this function by filling in the blanks below.

Minimum windows washed $\leq w \leq$ Maximum windows washed

Practical Domain: _____________ $\leq w \leq$ _____________

c) Identify the practical range of this function by filling in the blanks below.

Minimum Cost $\leq C(w) \leq$ Maximum Cost

Practical Range: _____________ $\leq C(w) \leq$ _____________

d) Enter the equation for C into the Y= part of your calculator. Then use the TABLE feature to complete the table below:

w	0	50	150	200
$C(w)$				

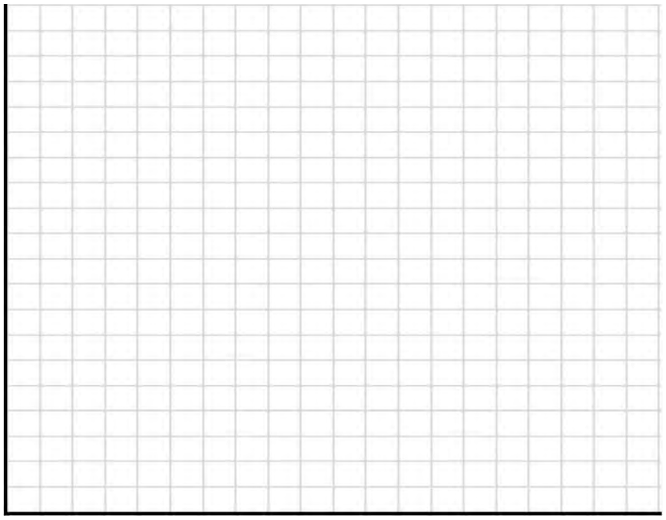
e) Use the TABLE to determine the value of $C(50)$. Circle the appropriate column in the table. Write a sentence explaining the meaning of your answer.

f) Use the TABLE to determine w when $C(w) = 45$. Circle the appropriate column. Write a sentence explaining the meaning of your answer.

g) Use your FUNCTION from part a) to determine the value of w when $C(w) = 45$. Set up the equation, $C(w) = 45$ then solve for the value of w.

30. Suppose the number of pizzas you can make in an 8 hour day is determined by the function $P(t) = 12t$ where P is the output (Pizzas made) and t is the input (Time in hours).

a) Graph this function using your calculator. [Go to Y= and type 12x into the Y1 slot. Then, press WINDOW and enter xmin = 0, xmax =8, ymin = 0, and ymax=96 then press GRAPH]. Show a good graph in the space below.



b) Use the Table feature of your graph and identify the first and last ordered pairs that are on the graph (based on the information above). [2nd>Graph will take you to the table]. Include both ordered pairs and function notation.

c) What is the INPUT quantity (including units) for this function? Name the smallest and largest possible input quantity then use this information to identify the PRACTICAL DOMAIN.

Input quantity (including units):______________________________

Practical domain:

Inequality notation:______________________

Interval notation:________________________

d) What is the OUTPUT quantity (including units) for this function? Name the smallest and largest possible output quantity then use this information to identify the PRACTICAL RANGE.

Output quantity (including units):______________________________

Practical range:

Inequality notation:______________________

Interval notation:_________________________

e) Find $P(3)$ and interpret its meaning in the context of the problem.

f) Find t so that $P(t) = 70$ and interpret its meaning in the context of the problem.

31. The life expectancy for males in the United States from the year 1900 until 2020 can be modeled by the function $L(x) = 0.27x + 48.3$, where L is the life expectancy and x is the number of years since 1900.

a) Which letter, L or x is used for input?

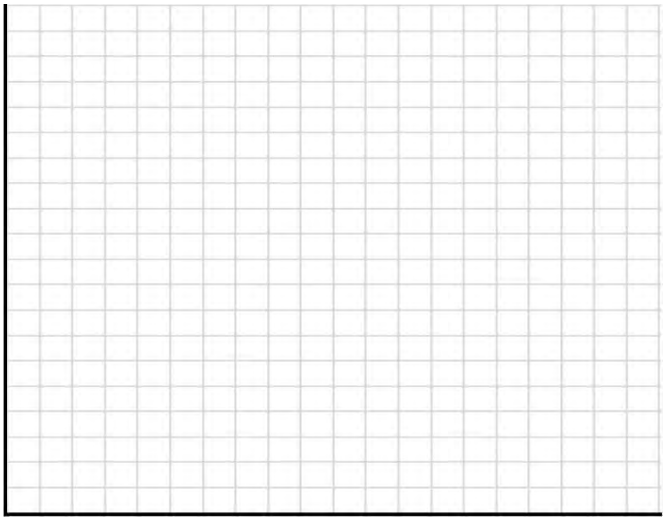
b) What does the INPUT represent? Include units.

c) Which letter, L or x, is used for output?

d) What does the OUTPUT represent? Include units.

e) Draw a neat, labeled and accurate sketch of this graph in the space below.

x	L(x)
0	
20	
40	
60	
80	
100	
120	



f) What is the practical domain of $L(x)$? Use proper inequality notation.

g) What is the practical range of $L(x)$? Use proper inequality notation.

h) What is the life expectancy of a man born in Iowa in 1950?

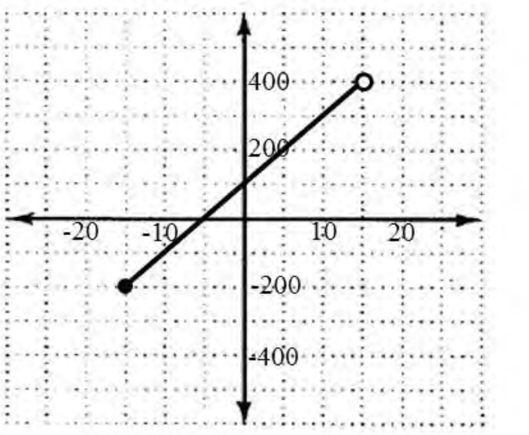
i) If a man is expected to live to the age of 60, approximate the year he was born. (Round to nearest year)?

Name:______________________ **Date:**________________

Lesson 1 Assessment

1. Let $r(a) = 4 - 5a$. Show all steps. Write each answer using function notation **and** as an ordered pair.
   a) Determine $r(-2)$.
   b) For what value of a is $r(a) = 19$?

2. The graph of $f(x)$ is given below. Use inequality notation.



a) Give the domain of $f(x)$:

b) Give the range of $f(x)$:

c) $f(0) =$ _________

d) $f(x) = 0$ when $x =$_________________

3. Consider the following table of values. Fill in the blanks below, and identify the corresponding ordered pairs.

x	–2	–1	0	1	2	3	4
g(x)	1	4	2	6	5	0	2

g(1) = _______, $g(x) = 1$ when $x =$ ________, $g(x) = 2$ when $x =$ _________________

4. The distance, *d* (in miles), of a car is a function of the time, *t* (in hours), given the car is traveling at a constant rate, $r = 45\ mph$. The driver is driving from Hartford, CT to New York City; a distance of 117 miles. The car's distance as a function of time is modeled by the equation $d(t) = 45t$.

a) Use the TABLE feature on your graphing calculator to complete the table below.

t	0	1	2	3
d(*t*)				

b) Determine $d(1.5)$. Write a sentence explaining the meaning of your answer.

c) For what values of *t* is *d*(*t*) = 0? Write a sentence explaining the meaning of your answer.

d) After how many hours of driving will the driver arrive at New York City? Show all work.

e) Determine the practical domain. Use inequality notation and include units.

f) Determine the practical range. Use inequality notation and include units.

g) Use your graphing calculator to generate a graph of *d*(*t*). Use the practical domain and range to determine a "good" viewing window. In the space below, sketch what you see on your calculator screen, and write down the viewing window you used.

Xmin=____________

Xmax=____________

Ymin=____________

Ymax=___________

Lesson 2 – Functions and Function Operations

As we continue to work with more complex functions it is important that we are comfortable with Function Notation, operations on Functions and operations involving more than one function. In this lesson, we study using proper Function Notation and then spend time learning how add, subtract, multiply and divide Functions, both algebraically and when the functions are represented with a tables or graphs. Finally, we take a look at a couple of real world examples that involve operations on functions.

Lesson Topics:

Section 2.1: Combining Functions

- Basic operations: Addition, Subtraction, Multiplication, and Division
- Multiplication Property of Exponents
- Division Property of Exponents
- Negative Exponents
- Operations on Functions in table form
- Operations on Functions in graph form

Section 2.2: Applications of Function Operations

- Cost, Revenue, and Profit

Section 2.3: Composition of Functions

- Evaluating Functions
- Composition of Functions in table form
- Composition of Functions in graph form

Section 2.4: Applications of Function Composition

Mini-Lesson 2

Section 2.1 – Combining Functions

Function notation can be expanded to include notation for the different ways we can combine functions as described below.

Basic Mathematical Operations
The basic mathematical operations are: addition, subtraction, multiplication, and division. When working with function notation, these operations will look like this:

Addition	Subtraction	Multiplication	Division
$f(x)+g(x)$	$f(x)-g(x)$	$f(x)\cdot g(x)$	$\frac{f(x)}{g(x)} \quad g(x)\neq 0$

Many of the problems we will work in this lesson are problems you may already know how to do. You will just need to get used to some new notation.

We will start with the operations of addition and subtraction.

Problem 1	**WORKED EXAMPLE – Adding and Subtracting Functions**

Given $f(x) = 2x^2 + 3x - 5$ and $g(x) = -x^2 + 5x + 1$.

a) Find $f(x) + g(x)$

$$f(x)+g(x)=\left(2x^2+3x-5\right)+\left(-x^2+5x+1\right)=2x^2+3x-5-x^2+5x+1=2x^2-x^2+3x+5x-5+1$$

$$f(x)+g(x)=x^2+8x-4$$

b) Find $f(x) - g(x)$

$$f(x)-g(x)=\left(2x^2+3x-5\right)-\left(-x^2+5x+1\right)=2x^2+3x-5+x^2-5x-1$$

$$=2x^2+x^2+3x-5x-5-1$$

$$f(x)-g(x)=3x^2-2x-6$$

c) Find $f(1) - g(1)$

$$f(1)-g(1)=\left[2(1)^2+3(1)-5\right]-\left[-(1)^2+5(1)+1\right]=(2+3-5)-(-1+5+1)=0-5$$

$$f(1)-g(1)=-5$$

Problem 2	MEDIA EXAMPLE – Adding and Subtracting Functions

Given $f(x) = 3x^2 + 2x - 1$ and $g(x) = x^2 + 2x + 5$:

a) Find $f(x) + g(x)$

b) Find $f(x) - g(x)$

Problem 3	YOU TRY – Adding and Subtracting Functions

Given $f(x) = x^2 + 4$ and $g(x) = x^2 + 1$, determine each of the following. Show complete work.

a) Find $f(2) + g(2)$

b) Find $f(x) - g(x)$

c) Find $f(2) - g(2)$

Function Multiplication and the Multiplication Property of Exponents

When multiplying functions, you will often need to work with exponents.
The following should be familiar to you and will come into play in the examples below:

MULTIPLICATION PROPERTY OF EXPONENTS

Let *m* and *n* be rational numbers.
To multiply powers of the same base, keep the base and add the exponents:

$$\boldsymbol{a^m \cdot a^n = a^{m+n}}$$

Problem 4	**WORKED EXAMPLE – Function Multiplication**

a) Given $f(x) = -8x^4$ and $g(x) = 5x^3$, find $f(x) \cdot g(x)$

$f(x) \cdot g(x) = (-8x^4)(5x^3)$

$= (-8)(5)(x^4)(x^3) = (-40)(x^{4+3})$

$f(x) \cdot g(x) = -40x^7$

Reorder using Commutative Property
Simplify using the Multiplication Property of Exponents
Final Result

b) Given $f(x) = -3x$ and $g(x) = 4x^2 - x + 8$, find $f(x) \cdot g(x)$

$f(x) \cdot g(x) = (-3x)(4x^2 - x + 8)$

$f(x) \cdot g(x) = -12x^3 + 3x^2 - 24x$

Apply the Distributive Property
Remember the rules of exp.
$(-3x)(4x^2) = (-3)(4)(x^1)(x^2)$
$= -12x^3$
Final Result

c) Given $f(x) = 3x + 2$ and $g(x) = 2x - 5$, find $f(x) \cdot g(x)$

$f(x) \cdot g(x) = (3x + 2)(2x - 5)$

$= (3x)(2x) + (3x)(-5) + (2)(2x) + (2)(-5)$

$= \mathit{FIRST} + \mathit{OUTER} + \mathit{INNER} + \mathit{LAST}$

$= (6x^2) + (-15x) + (4x) + (-10)$

$f(x) \cdot g(x) = 6x^2 - 11x - 10$

Use FOIL
Remember the rules of exp.
$(3x)(2x) = (3)(2)(x)(x)$
$= 6x^2$
Combine Like Terms
Final Result

Problem 5	**MEDIA EXAMPLE – Function Multiplication**

Given $f(x) = 3x + 2$ and $g(x) = 2x^2 + 3x + 1$, find $f(x) \cdot g(x)$

Problem 6	**YOU TRY – Function Multiplication**

For each of the following, find $f(x) \cdot g(x)$

a) $f(x) = 3x - 2$ and $g(x) = 3x + 2$

b) $f(x) = 2x^2$ and $g(x) = x^3 - 4x + 5$

c) $f(x) = 4x^3$ and $g(x) = -6x$

d) $f(x) = 2x - 1$ and $g(x) = -3x^2 - 2x + 3$

Function Division and the Division Property of Exponents

When dividing functions, you will also need to work with exponents of different powers. The following should be familiar to you and will come into play in the examples below:

DIVISION PROPERTY OF EXPONENTS

Let m, n be rational numbers. To divide powers of the same base, keep the base and subtract the exponents.

$\frac{a^m}{a^n} = a^{m-n}$ where a ≠ 0, m,n are integers.

Problem 7	**WORKED EXAMPLE – Function Division**

For each of the following, find $\frac{f(x)}{g(x)}$. Use only positive exponents in your final answer.

a) $f(x) = 15x^{15}$ and $g(x) = 3x^9$

$$\begin{aligned}\frac{f(x)}{g(x)} &= \frac{15x^{15}}{3x^9} \\ &= 5x^{15-9} \\ &= 5x^6\end{aligned}$$

NEGATIVE EXPONENTS

If $a \neq 0$ and n is a rational number, then $a^{-n} = \frac{1}{a^n}$

b) $f(x) = -4x^5$ and $g(x) = 2x^8$

$$\begin{aligned}\frac{f(x)}{g(x)} &= \frac{-4x^5}{2x^8} \\ &= -2x^{5-8} \\ &= -2x^{-3} \\ &= -\frac{2}{x^3}\end{aligned}$$

Problem 8 MEDIA EXAMPLE – Function Division

For each of the following, determine $\frac{f(x)}{g(x)}$. Use only positive exponents in your final answer.

a) $f(x) = 10x^4 + 3x^2$ and $g(x) = 2x^2$

b) $f(x) = -12x^5 + 8x^2 + 5$ and $g(x) = 4x^2$

Problem 9 YOU TRY – Function Division

For each of the following, determine $\frac{f(x)}{g(x)}$. Use only positive exponents in your final answer.

a) $f(x) = 25x^5 - 4x^7$ and $g(x) = -5x^4$

b) $f(x) = 20x^6 - 16x^3 + 8$ and $g(x) = -4x^3$

Functions can be presented in multiple ways including: equations, data sets, graphs, and applications. If you understand function notation, then the process for working with functions is the same no matter how the information if presented.

Problem 10	**MEDIA EXAMPLE – Working with Functions in Table Form**

Functions $f(x)$ and $g(x)$are defined in the tables below. Find a – e below using the tables.

x	–3	–2	0	1	4	5	8	10	12
$f(x)$	8	6	3	2	5	8	11	15	20

x	0	2	3	4	5	8	9	11	15
$g(x)$	1	3	5	10	4	2	0	–2	–5

a) $f(1) =$

b) $g(9) =$

c) $f(0) + g(0) =$

d) $g(5) - f(8) =$

e) $f(0) \cdot g(3) =$

Problem 11	**YOU TRY – Working with Functions in Table Form**

Given the following two tables, complete the third table. Show work in the table cell for each column. The first one is done for you.

x	0	1	2	3	4
$f(x)$	4	3	–2	0	1

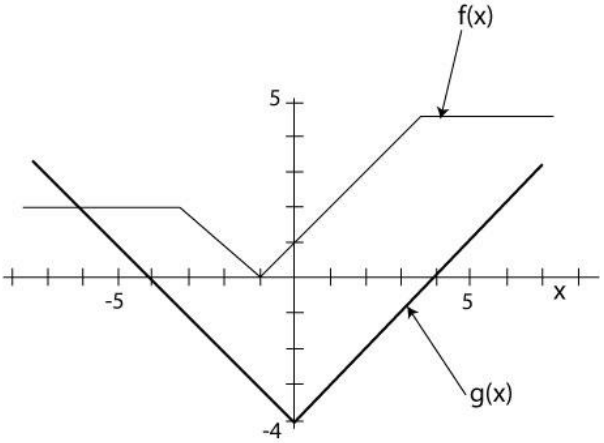
x	0	1	2	3	4
$g(x)$	6	–3	4	–2	2

x	0	1	2	3	4
$f(x)+g(x)$	$f(0)+g(0)$ $=4+6$ $=10$	$f(1)+g(1)$			

If you remember that graphs are just infinite sets of ordered pairs and if you do a little work ahead of time (as in the example below) then the graphing problems are a lot easier to work with.

Problem 12	**YOU TRY – Working with Functions in Graph Form**

Use the graph to determine each of the following. The graph of g is the graph in bold.



Complete the following ordered pairs from the graphs above. Use the information to help you with the problems below. The first ordered pair for each function has been completed for you.

f: (–7, 2), (–6,), (–5,), (–4,), (–3,), (–2,), (–1,), (0,), (1,), (2,), (3,), (4,), (5,), (6,), (7,)

g: (–7, 3), (–6,), (–5,), (–4,), (–3,), (–2,), (–1,), (0,), (1,), (2,), (3,), (4,), (5,), (6,), (7,)

a) $g(4) =$ ________________________

b) $f(2) =$____________________

c) $g(0) =$ ________________________

d) $f(-6) =$ ____________________

e) If $f(x) = 0, x =$ ____________________

f) If $g(x) = 0, x =$ ____________________

g) If $f(x) = 1, x =$____________________

h) If $g(x) = -4, x =$____________________

i) $f(-1) + g(-1) =$____________________

j) $g(-6) - f(-6) =$____________________

k) $f(1) * g(-2) =$____________________

l) $\dfrac{g(6)}{f(-1)} =$____________________

Section 2.2 – Applications of Function Operations

One of the classic applications of function operations is the forming of the Profit function, P(x) by subtracting the cost function, C(x), from the revenue function, R(x) as shown below.

Profit = Revenue – Cost

Given functions $P(x)$ = Profit, $R(x)$ = Revenue, and $C(x)$ = Cost:

$$P(x) = R(x) - C(x)$$

Problem 13	**MEDIA EXAMPLE – Cost, Revenue, Profit**

A local courier service estimates its monthly operating costs to be $1500 plus $0.85 per delivery. The service generates revenue of $6 for each delivery. Let x = the number of deliveries in a given month.

a) Write a function, $C(x)$, to represent the monthly costs for making x deliveries per month.

b) Write a function, $R(x)$, to represent the revenue for making x deliveries per month.

c) Write a function, $P(x)$, that represents the monthly profits for making x deliveries per month.

d) Determine algebraically the break-even point for the function $P(x)$ and how many deliveries you must make each month to begin making money. Show complete work. Write your final answer as a complete sentence.

e) Determine the break-even point graphically by solving the equation $P(x) = 0$. Explain your work and show the graph with appropriate labels. Write your final answer as a complete sentence.

Problem 14	**YOU TRY – Cost, Revenue, Profit**

February is a busy time at Charlie's Chocolate Shoppe! During the week before Valentine's Day, Charlie advertises that his chocolates will be selling for \$1.80 a piece (instead of the usual \$2.00 each). The fixed costs to run the Chocolate Shoppe total \$450 for the week, and he estimates that each chocolate costs about \$0.60 to produce. Charlie estimates that he can produce up to 3,000 chocolates in one week.

a) Write a function, $C(n)$, to represent Charlie's total costs for the week if he makes n chocolates.

b) Write a function, $R(n)$, to represent the revenue from the sale of n chocolates during the week before Valentine's Day.

c) Write a function, $P(n)$, that represents Charlie's profit from selling n chocolates during the week before Valentine's Day. Show complete work to find the function.

d) Write a complete sentence to interpret the meaning of the statement $P(300) = -90$.

e) Determine the Practical Domain and Practical Range for $P(n)$, then use that information to define an appropriate viewing window for the graph of $P(n)$. Sketch the graph from your calculator in the space provided.

Practical Domain:

Practical Range:

f) How many chocolates must Charlie sell in order to break even? Show complete work. Write your final answer as a complete sentence. **Mark the break-even point on the graph above.**

Section 2.3 – Composition of Functions

Composition of Functions

Function Composition is the process by which the OUTPUT of one function is used as the INPUT for another function. Two functions $f(x)$ and $g(x)$ can be composed as follows:

$f(g(x))$, where the function $g(x)$ is used as the INPUT for the function $f(x)$.

OR

$g(f(x))$, where the function $f(x)$ is used as the INPUT for the function $g(x)$.

Problem 15	**WORKED EXAMPLE – Composition of Functions**

Let $f(x) = 5 - x$ and $g(x) = 3x + 4$. Evaluate each $f(g(x))$, $g(f(x))$, and $f(g(7))$.

To evaluate $f(g(7))$, always start with the "inside" function. In this case, $g(7)$.

$$\begin{aligned} g(7) &= 3(7)+4 \\ &= 21+4 \\ &= 25 \end{aligned}$$

Then plug this result (output) into $f(x)$.

$$\begin{aligned} f(g(7)) &= f(25) \\ &= 5-(25) \\ &= -20 \end{aligned}$$

$$\begin{aligned} f(g(x)) &= f(3x+4) \\ &= 5-(3x+4) \\ &= 5-3x-4 \\ &= -3x+1 \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(5-x) \\ &= 3(5-x)+4 \\ &= 15-3x+4 \\ &= -3x+19 \end{aligned}$$

Problem 16	**MEDIA EXAMPLE – Composition of Functions**

Let $A(x) = 2x + 1$ and $B(x) = 3x - 5$. Evaluate each of the following.

$A(B(x)) =$ $B(A(x)) =$

$A(B(4)) =$ $B(A(4)) =$

Problem 17	**YOU TRY– Composition of Functions**

Let $f(x) = 4 - 3x$ and $g(x) = x - 8$. Evaluate each of the following.

a) $f(g(x)) =$ b) $g(f(5)) =$

Problem 18	**MEDIA EXAMPLE – Composition of Functions Given in Table Form**

The functions $f(x)$ and $g(x)$ are defined by the tables below.

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	4	11	10	8	6	5	8	2	6	9

x	1	2	3	4	5	6	7	8	9	10
$g(x)$	3	8	4	10	2	5	11	0	4	1

$f(g(5)) =$

$f(g(1)) =$

$g(f(4)) =$

$f(f(1)) =$

Problem 19	**YOU TRY – Composition of Functions Given in Table Form**

The functions A(x) and B(x) are defined by the tables below.

x	A(x)
0	5
1	7
2	3
3	6
4	1
5	2
6	11
7	3

x	B(x)
0	7
1	0
2	4
3	2
4	6
5	1
6	3
7	15

a) A(B(4)) = ____________________________

b) B(A(1)) = ____________________________

d) B(A(7)) = ____________________________

d) A(B(1)) = ____________________________

Section 2.4 – Applications of Function Composition

Problem 20	**MEDIA EXAMPLE – Applications of Function Composition**

Lisa makes $18 per hour at her new part-time job.

a) Write a function, I, to represent Lisa's income for the week if she works h hours. Complete the table below.

$I(h) =$ ________________________________

h	5	10	15	20
$I(h)$				

b) Lisa puts 10% of her salary in her bank savings every week and $10 into her piggy bank for a rainy day. Write a function, S, to represent the total amount of money she saves each week if her income is I dollars. Complete the table below.

$S(I) =$ ________________________________

I	90	180	270	360
$S(I)$				

c) Using the information above, write a formula for $S(I(h))$ and complete the table below.

$S(I(h)) =$ ________________________________

h	5	10	15	20
$S(I(h))$				

d) What information does the function $S(I(h))$ provide in this situation? Be sure to identify the input and output quantities.

e) Interpret the meaning of the statement $S(I(10)) = 28$. Include all appropriate units.

Problem 21	**YOU TRY – Applications of Function Composition**

A resort hotel in Scottsdale, AZ charges $1800 to rent a reception hall, plus $58 per person for dinner and open bar. The reception hall can accommodate up to 200 people.

a) Write a function, *T,* to represent the total cost to rent the reception hall if *n* people attend the reception.

$T(n) =$ ________________________

b) During the summer months, the hotel offers a discount of 15% off the total bill, *T*. Write a function, *D,* to represent the discounted cost if the total bill was $*T*.

$D(T)=$ ________________________

c) Using the information above, write a formula for $D(T(n))$ and complete the table below.

$D(T(n)) =$ _________________________________

n	0	50	100	150	200
$D(T(n))$					

d) What information does the function $D(T(n))$ provide in this situation? Be sure to identify the input and output quantities.

e) Interpret the meaning of the statement $D(T(100)) = 6460$. Include all appropriate units.

f) Determine the maximum number of people that can attend the reception for $5,000 (after the discount is applied)?

Lesson 2 Practice Problems

Round to two decimal places unless stated otherwise.

Section 2.1: Combining Functions

1. Let $f(x) = -3x + 2$ and $g(x) = x^2 + 4x - 7$.

   Find the following and simplify your result.

   a) $f(4) + g(4) =$

   b) $g(-3) - f(-3) =$

   c) $f(2) \cdot g(2) =$

   d) $\frac{g(0)}{f(0)} =$

2. Let $f(x) = 2x - 4$ and $g(x) = x^2 - 9$. Find the following. Simplify your answers.

   a) $f(x) + g(x) =$

   b) $g(x) - f(x) =$

   c) $f(x) \cdot g(x) =$

   d) $\frac{g(x)}{f(x)} =$

3. Add, subtract and multiply the following functions. Simplify your answers.

   a) $f(x) = -4x + 7$ and $g(x) = -3x$

   $f(x) + g(x) =$

   $f(x) - g(x) =$

   $f(x) \cdot g(x) =$

   $g(x) - f(x) =$

b) $f(x) = -x + 2$ and $g(x) = -3x + 7$

$f(x) + g(x) =$

$f(x) - g(x) =$

$f(x) \cdot g(x) =$

$g(x) - f(x) =$

c) $f(x) = 3x^2 + 4x + 2$ and $g(x) = 6x + 1$

$f(x) \cdot g(x) =$

$f(4) + g(-1) =$

4. Simplify each of the following functions. Use only positive exponents in your final answer.

a) $f(x) = 32x^4 - 3x^7 \ \ and \ \ g(x) = 6x^4$

$\frac{f(x)}{g(x)} =$

b) $f(x) = 48x^9 - 16x^3 + 4 \ and \ g(x) = -8x^3$

$\frac{f(x)}{g(x)} =$

5. Use the tables of the functions below, find the following.

x	-2	-1	0	1	2
$f(x)$	-3	0	4	9	15

x	-2	-1	0	1	2
$g(x)$	12	8	1	-3	-5

a) $f(2)+g(2)=$

b) $g(-1)-f(-1)=$

c) $f(0)\cdot g(0)=$

d) $\dfrac{g(1)}{f(1)}=$

6. Functions *f(x)* and *g(x)* are defined in the tables below. Use the tables to evaluate the functions below.

x	-3	-2	0	1	4	5	8	10	12
$f(x)$	8	6	3	2	5	8	11	15	20

x	0	2	3	4	5	8	9	11	15
$g(x)$	1	3	5	10	4	2	0	-2	-5

a) $f(5)=$

b) $g(5)=$

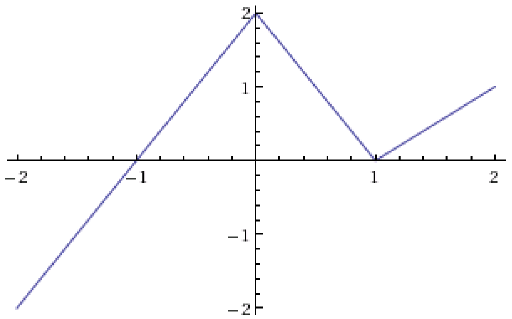
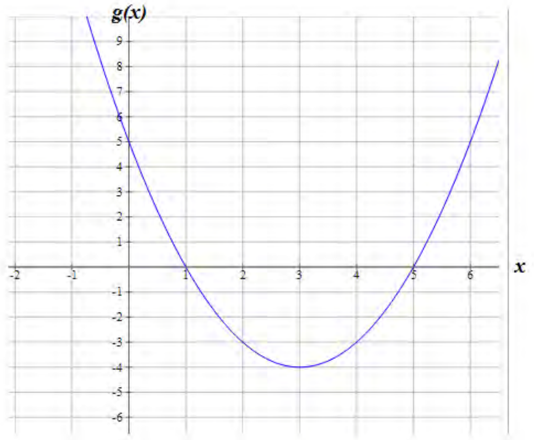
c) $f(5)+g(5)=$

d) $f(0)-g(0)=$

e) $f(8)\cdot g(8)=$

f) $f(4)\cdot g(0)=$

7. Use the graph to determine each of the following. Assume integer answers.

f(x)	*g(x)*
	

a) $f(0)+g(0)=$

b) $f(1)-g(3)=$

c) $f(-2)\cdot g(5)=$

d) $f(-1)\cdot g(2)=$

8. Functions f and g are defined below. Use those functions to evaluate the problems below.

$f=\{(-3,4),(-2,6),(-1,8),(0,6),(1,-2)\}$

$g=\{(-1,8),(0,2),(4,3),(8,4)\}$

a) $f(-2)+g(0)=$

b) $f(1)-g(4)=$

c) $f(0)\cdot g(0)=$

d) $f(-1)\cdot g(8)=$

Section 2.2: Applications of Function Operations

9. The function $E(n)$ represents Ellen's budgeted monthly expenses for the first half of the year 2013. In the table, $n = 1$ represents January 2013, $n = 2$ February 2013, and so on.

n	1	2	3	4	5	6
$E(n)$	2263	2480	2890	2263	2352	2550

The function $L(n)$ shown in the table below represents Ellen's monthly income for the first half of the year 2013. In the table, $n = 1$ represents January 2013, $n = 2$ February 2013, and so on.

n	1	2	3	4	5	6
$L(n)$	2850	2850	2850	2850	2850	2850

a) At the end of each month, Ellen puts any extra money into a savings account. The function $S(n)$ represents the amount of money she puts into savings each month. Using the information above, complete the following table for the function $S(n)$.

n	1	2	3	4	5	6
$S(n)$						

b) Her goal is to save enough money to take a trip to Hawaii in July, 2013. She estimates that the trip will cost \$2000. Will she be able to save up enough money to go to Hawaii in July? If so, how much extra money will he have to spend while she is there? If not, how much more does she need to earn?

10. Maria and Todd are organizing the 20 year reunion for their high school. The high school alumni association has given them \$1000 for the event. They talk to the local caterer and find out the following:

- It will cost \$15 per person plus a \$50 setup fee to provide food for the event.
- It will cost \$3 per person plus an \$80 setup fee to rent the Meeting Hall at the local Holiday Motel.

To help determine the costs, they come up with the following functions:

- The cost for food is \$50 + \$15 per person. $\boldsymbol{F(x) = 15x + 50}$
- The cost for the Hall is \$80 + \$3 per person $\boldsymbol{H(x) = 3x + 80}$

In addition, they decide to charge each person \$5 to get in the door. This can be modeled by the following function:

- Income for the event is \$1000 from the alumni + \$5 per person. $\boldsymbol{I(x) = 5x + 1000}$

Given this information, answer the following questions. Show how you use the functions to calculate the answers. And give your final answers in complete sentences.

If 400 people attend the event:

a) How much will it cost for food?

b) How much will it cost to rent the Meeting Hall?

c) How much will it cost for food AND to rent the Meeting Hall? <u>Show how you use the functions to calculate this.</u> Hint: $F(400) + H(400)$

d) The final bill for the event is found by subtracting the costs from the income. What would the final bill for the event be?

e) <u>Challenge question</u>. How many people can attend if the costs have to equal the income?

11. Leonard has started a new business making cartoon bedspreads. His monthly expenses are $1322. Each bedspread costs $8.50 to produce.

a) Complete the table below showing Leonard's business costs as a function of the number of bedspreads he makes.

n *(number of bedspreads)*	0	100	200	300	400
C(*n*) *(Cost of n bedspreads)*					

b) Leonard is selling each bedspread for $17.50. Complete the table below showing Leonard's revenue as a function of the number of bedspreads he sells.

n *(number of bedspreads)*	0	100	200	300	400
R(*n*) *(Revenue for n bedspreads)*					

c) The profit from Leonard's business can be found by subtracting the cost function from the revenue function. Complete the table below showing Leonard's profit as a function of the number of bedspreads he sells.

n *(number of bedspreads)*	0	100	200	300	400
P(*n*) *(Profit for n bedspreads)*					

d) Using the information from parts a) through c), create algebraic functions for C, R and P.

$C(n) =$

$R(n) =$

$P(n) =$

e) Using the table from part c), make a rough estimate for the number of bedspreads Leonard needs to sell for his business to break even. (Breaking even means profit = 0)

f) Using your formula for profit, P, determine the exact number of bedspreads Leonard needs to sell for his business to break even. (Breaking even means profit = 0)

Section 2.3: Composition of Functions

12. Let $f(x)=4x-2$ and $g(x)=-2x+5$. Evaluate each of the following.

a) $f\left(g(-2)\right)$

b) $g\left(f(-2)\right)$

c) $f\left(g(4)\right)$

d) $g\left(f(0)\right)$

13. Let $s(t)=t^2-2$ and $q(t)=-2t-3$. Evaluate each of the following.

a) $s\left(q(-2)\right)=$

b) $q\left(s(-2)\right)=$

c) $s\left(q(-1)\right)=$

d) $q\left(s(0)\right)=$

14. Let $f(x) = 4x - 2$ and $g(x) = -2x + 5$. Find each of the following. Simplify your answers.

a) $f(g(x)) =$

b) $g(f(x)) =$

15. Let $s(t) = t^2 - 7$ and $q(t) = t + 4$. Find each of the following. Simplify your answers.

a) $s(q(t)) =$

b) $q(s(t)) =$

16. Using the functions $f(x)$ and $g(x)$ defined by the tables below, evaluate the compositions.

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	2	9	4	1	6	5	8	1	6	10

x	1	2	3	4	5	6	7	8	9	10
$g(x)$	4	11	10	8	6	5	8	2	6	9

a) $f(g(3)) =$

b) $f(g(7)) =$

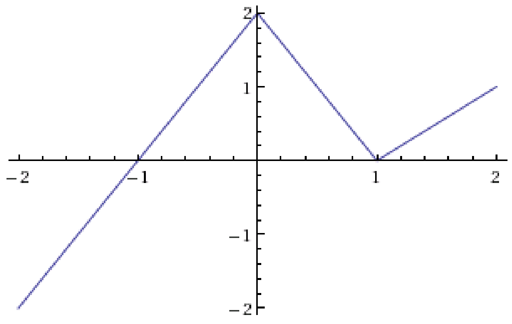
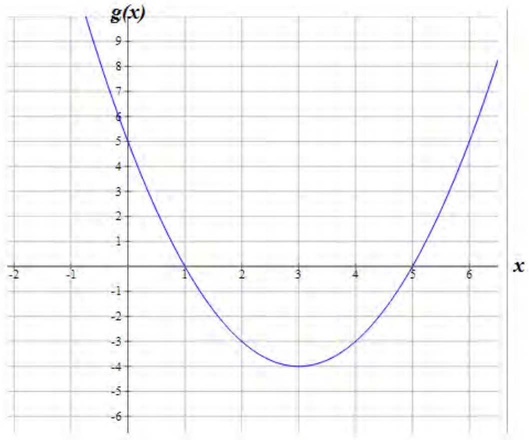
c) $g(f(9)) =$

d) $f(f(8)) =$

e) $g(g(5)) =$

f) $g(f(10)) =$

17. Using the functions $f(x)$ and $g(x)$ defined by the graphs below, evaluate the compositions.

$f(x)$	$g(x)$
	

a) $f(g(1)) =$

b) $g(f(0)) =$

c) $g(f(-1)) =$

d) $f(g(5)) =$

Section 2.4: Applications of Function Composition

18. Raj likes playing video games. He earns 27 tokens every hour he plays.

a) Write a function, T, which represents the number of tokens Raj earns for the week if he plays h hours. Also complete the table below.

$T(h) =$ ________________________________

h	10	20	30	40
$T(h)$				

b) Raj can use his tokens to buy additional plays on the game *Bollywood Dance*. Each *Bollywood Dance* game costs 90 tokens. Write a function, B, which represents the number of games of *Bollywood Dance* that Raj can buy in a week if he earns T tokens. Also complete the table below.

$\mathrm{B}(T) =$ ________________________________

T	270	540	810	1080
$B(T)$				

c) Using the information above, write a formula for $B(T(h))$ and complete the table below.

$B(T(h)) =$ ________________________________

h	10	20	30	40
$B(T(h))$				

d) Determine the largest number of Bollywood Dance games Raj can buy if he plays 23 hours in a week.

e) Interpret the meaning of the statement $B(T(15)) = 5.0625$. Include all appropriate units.

19. A waterpark charges $1200 to rent the park per day, plus $37 for each person who attends. The waterpark can accommodate up to 200 people.

a) Write a function, T, to represent the total cost to rent the waterpark if n people attend.

$T(n) =$ ________________________

b) During the winter months, the waterpark offers a discount of 12% off the total bill, T. Write a function, D, to represent the discounted cost if the total bill was $\$T$.

$D(T) =$ ________________________

c) Using the information above, write a formula for $D(T(n))$ and complete the table below.

$D(T(n)) =$ __________________________________

n	0	50	100	150	200
$D(T(n))$					

d) What information does the function $D(T(n))$ provide in this situation? Be sure to identify the input and output quantities.

e) Interpret the meaning of the statement $D(T(75)) = 3498$. Include all appropriate units.

f) Determine the maximum number of people that can attend the waterpark for $5,000 (after the discount is applied)?

Lesson 2 Assessment

1. If possible, simplify each of the following by combining like terms or using properties of exponents.

   a) $2n^5 + 3n^5 =$ _______________ b) $2n^5 \cdot 3n^5 =$ _______________

   c) $3n^3 + 3n^5 =$ _______________ d) $3n^3 \cdot 3n^5 =$ _______________

2. The functions *A* and *B* are defined by the following tables

X	–3	–2	0	1	4	5	8	10	12
$A(x)$	8	6	3	2	5	8	11	15	20

X	0	2	3	4	5	8	9	11	15
$B(x)$	1	3	5	10	4	2	0	–2	–5

Determine the values for each of the following.

a) $B(3)=$_________ b) $A(8)=$_________ c) $A(0)+B(0)=$_______

d) $A(8) - B(8)=$_________ e) $A(4) \cdot B(4)=$_________ f) $\dfrac{A(5)}{B(5)} =$________

3. Let $p(x) = x^2 + 2x + 3$ and $r(x) = x - 5$. Determine each of the following. Show all work. Box your answers.

   a) $p(x) - r(x)=$ b) $p(0) \cdot r(0)=$

   c) $p(-2) + r(-2)=$ d) $r(7) - p(7)=$

   e) $p(x) \cdot r(x) =$

4. If $f(x) = 4x^3 - 2x^2 + 8x$ and $g(x) = 2x$, divide $\dfrac{f(x)}{g(x)}$.

4. A resort hotel charges $2200 to rent a reception hall, plus $65 per person for dinner and open bar. The reception hall can accommodate up to 200 people.

   a) Write a function, *T,* to represent the total cost to rent the reception hall if n people attend the reception.

   $T(n) =$ ______________________

   b) Identify the practical domain and practical range of this function (include units).

   Practical Domain:_____________ $\leq n \leq$ _______________

   Practical Range:______________ $\leq T(n) \leq$ ______________

   c) During the summer months, the hotel offers a discount of 10% off the total bill, *T*. Write a function, *D,* to represent the discounted cost if the total bill was $*T*.

   $D(T)=$ ______________________

   d) Using the information above, write a formula for D($T(n)$) and complete the table below.

   D($T(n)$) = ______________________________

n	0	50	100	150	200
D($T(n)$)					

   e) What information does the function D($T(n)$) provide in this situation? Be sure to identify the input and output quantities.

   f) Interpret the meaning of the statement D($T(80)$) = 6660. Include all appropriate units.

   g) Determine the maximum number of people that can attend the reception for $10,000 (after the discount is applied)?

Lesson 3 – Linear Equations and Functions

The first Function that we are going to investigate is the Linear Function. This is a good place to start because with Linear Functions, the average rate of change is constant and no exponents are involved. Before we begin working with Linear Functions, though, we need to review the characteristics of Linear Equations and operations on Linear Equations.

Lesson Topics:

Section 3.1: Linear Equations and Functions

- Slope
- Slope-Intercept form of the equation of a line, $y = mx + b$
- Vertical Intercepts (y-int)
- Horizontal Intercepts (x-int)

Section 3.2: Graphs of Linear Functions

- Graph by plotting points
- Graph using slope
- Graph using intercepts

Section 3.3: Horizontal and Vertical Lines

- Equations of horizontal and vertical lines
- Graphs of horizontal and vertical lines

Section 3.4: Writing the Equation of a Line

- Linear equations from graphs
- Applications of linear equations

Mini-Lesson 3

Section 3.1 – Linear Equations and Functions

The topic of linear equations should be at least slightly familiar to students starting Intermediate Algebra. The basics are covered here with reminders of important ideas and concepts that will be heavily utilized in the next lesson.

Slope

Slope is a measure of steepness and direction for a given line. It is denoted by the letter *m*. Given any two points, (x_1, y_1), (x_2, y_2), on a line, the slope is determined by computing the following ratio:

$$m = \frac{\text{Change in Output}}{\text{Change in Input}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x}$$

Note: If the slope is negative, then the line decreases from left to right.

If slope is positive, then the line increases from left to right.

If the slope is zero, then the line is horizontal (constant)

A vertical line has no slope (the slope is undefined).

Problem 1	**WORKED EXAMPLE – Determine Slope of a Linear Equation / Function**

Find the slope of the line through the given points, then determine if the line is increasing, decreasing, horizontal, or vertical.

a) (2, –5) and (–3, 4).

$$\begin{aligned} m &= \frac{4-(-5)}{-3-(2)} \\ &= \frac{4+5}{-5} \\ &= \frac{9}{-5} \\ &= -\frac{9}{5} \end{aligned}$$

Decreasing

b) (–2, –4) and (4, 8)

$$\begin{aligned} m &= \frac{8-(-4)}{4-(-2)} \\ &= \frac{8+4}{4+2} \\ &= \frac{12}{6} \\ &= 2 \end{aligned}$$

Increasing

c) (2, 5) and (8, 5)

$$\begin{aligned} m &= \frac{5-5}{8-2} \\ &= \frac{0}{6} \\ &= 0 \end{aligned}$$

Horizontal (Constant)

Problem 2	**YOU TRY – Determine Slope of a Linear Equation/Function**

Find the slope of the line through the given points, then determine if the line is increasing, decreasing, horizontal, or vertical.

A) (5, –2) and (–3, 4).

B) (6, 2) and (4, –6)

SLOPE-INTERCEPT form for the equation of a line.

A LINEAR EQUATION is an equation that can be written in the form:

$$y = mx + b$$

with slope, $m \neq 0$, and Vertical Intercept (y-int) (0, *b*).

Using function notation, the equation of a line can be written as $f(x) = mx + b$.
DOMAIN: $(-\infty, \infty)$ (All real numbers)
RANGE: $(-\infty, \infty)$ (All real numbers)

Vertical Intercept (y-int) (0, *b*)

The Vertical Intercept (y-int) is the special ordered pair with coordinates (0, *b*). The input value is 0, and the resulting output is *b*.

The Vertical Intercept (y-int) is often used to help when graphing a linear equation and/or to determine the initial output value in a practical application.

There are 3 main methods for finding the Vertical Intercept (y-int) of a linear equation/function.

Method 1: Read the value of *b* from $y = mx + b$ or $f(x) = mx + b$ form.

Method 2: Solve for y when $x = 0$

Method 3: Evaluate $f(0)$.

Problem 3	**WORKED EXAMPLE – Determine Vertical Intercept (y-int) for a Linear Equation**

Example 1: Find the Vertical Intercept (y-int) for the equation $y = 2x - 5$.

This equation is written in the form $y = mx + b$. Here, $b = -5$.

Therefore, (using **Method 1**) the Vertical Intercept (y-int) is (0, –5).

Example 2: Find the Vertical Intercept (y-int) for the equation $y = 2x - 5$.

Using **Method 2**, set x to 0 and solve for y.

$$\begin{aligned} y &= 2(0) - 5 \\ y &= 0 - 5 \\ y &= -5 \end{aligned}$$

The Vertical Intercept (y-int) is (0, –5)

Example 3: Find the Vertical Intercept (y-int) of the linear function $f(x) = 2x - 5$.

In this example, use **Method 3** to evaluate $f(0)$.

$$\begin{aligned} f(0) &= 2(0) - 5 \\ &= 0 - 5 \\ &= -5 \end{aligned}$$

$f(0) = -5$, therefore the Vertical Intercept (y-int) is (0, –5)

Problem 4	**MEDIA EXAMPLE – Determine Slope and Vertical Intercept (y-int)**

Complete the problems below.

Equation	$f(x) = mx + b$ form	Slope / Behavior	Vertical Intercept (y-int)
a) $y = -2x + 5$			
b) $y = 2 - x$			
c) $y = \frac{3}{4}x + 2$			
d) $y = 4x$			
e) $y = -6$			
f) $y = x$			

Horizontal Intercept (x-int) (*a*, 0)

The Horizontal Intercept (x-int) is the special ordered pair with coordinates $(a, 0)$. The value a is the input value that results in an output of 0.

The Horizontal Intercept (x-int) is often used to help when graphing a linear equation and/or to determine the final input value in a practical application.

Problem 5	**MEDIA EXAMPLE – Find The Horizontal Intercept (x-int)**

For each of the following problems, determine the Horizontal Intercept (x-int) as an ordered pair. Write the answer as a fraction and a decimal rounded to two decimal places.

a) $y = –2x + 5$

b) $f(x) = 2 – x$

c) $g(x)=\frac{3}{4}x+2$

d) $y = 4x$

e) $f(x) = –6$

f) $y = x$

Problem 6	**WORKED EXAMPLE – Find The Horizontal Intercept (x-int) for a Linear Equation**

Find the Horizontal Intercept (x-int) for the equation $y = 2x - 5$.

Replace the value of y with 0 then solve for the value of x.

$$0 = 2x - 5$$

$$5 = 2x$$

$$\frac{5}{2} = x$$

The Horizontal Intercept (x-int) is $\left(\frac{5}{2}, 0\right)$

Problem 7	**YOU TRY – Find The Horizontal Intercept (x-int) for a Linear Equation/Function**

Complete the table below. Write intercepts as ordered pairs.
Write the answer as a fraction and a decimal rounded to two decimal places.

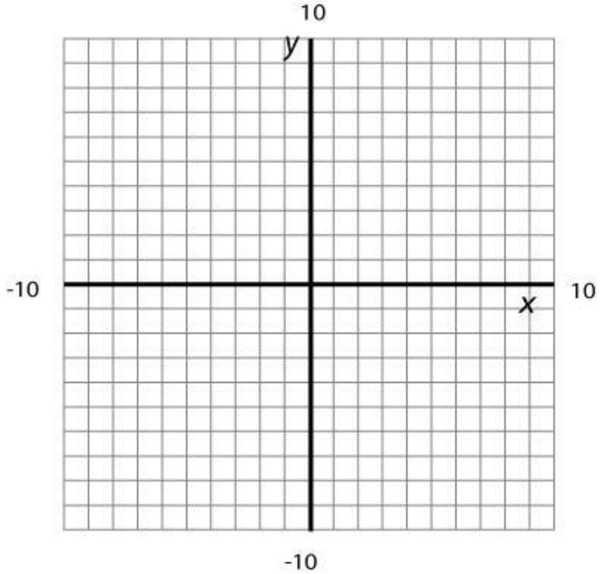
Equation	Slope / Behavior	Vertical Intercept (y-int)	Horizontal Intercept (x-int)
a) $f(x) = 6 - 4x$			
b) $y = 3x$			
c) $y = \frac{3}{5}x - 8$			

Section 3.2 – Graphs of Linear Functions

Problem 8 **MEDIA EXAMPLE – Graphing a Linear Equation by Plotting Points**

Graph the equation $f(x) = -2x + 6$

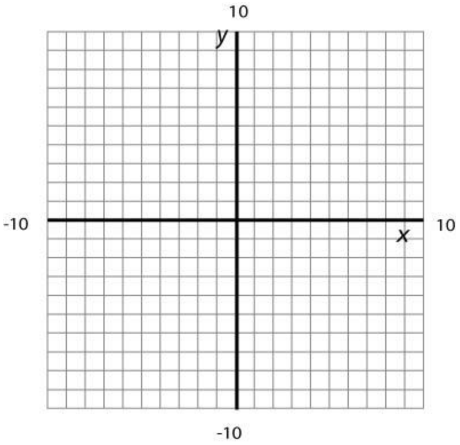
x	$f(x)$	Ordered Pair



Problem 9 **MEDIA EXAMPLE – Using the SLOPE to Graph a Linear Equation**

$$SLOPE = m = \frac{\text{Change in Output}}{\text{Change in Input}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x}$$

Draw an accurate graph for $y = -2x + 6$. Identify at least two additional points on the line, and label them on your graph.



Problem 10	**WORKED EXAMPLE – Use the Intercepts to Graph a Linear Equation**

Graph the equation $y = -2x + 6$ by plotting the intercepts on the graph.

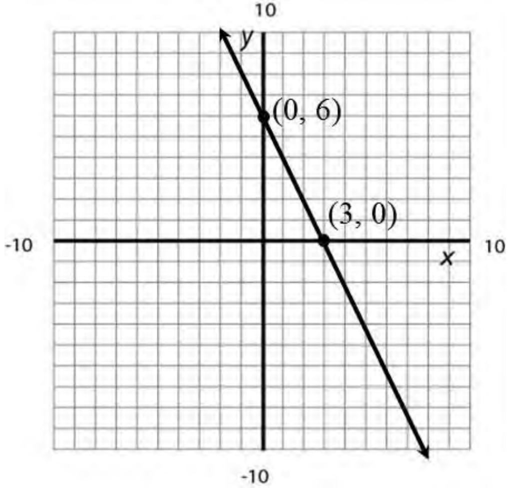
Vertical Intercept (y-int): This equation is written in the form $y = mx + b$, so the vertical intercept is (0, 6).

Horizontal Intercept (x-int): Set y to 0 and solve for x.

$$\begin{aligned} y &= -2x + 6 \\ 0 &= -2x + 6 \\ -6 &= -2x \\ 3 &= x \end{aligned}$$

So the Horizontal Intercept (x-int) is (3, 0).

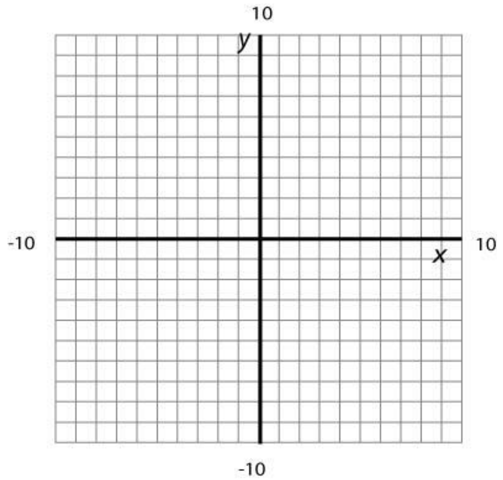
PLOT and LABEL the intercepts on the graph then connect them to draw your line.



Problem 11	**YOU TRY – Draw Graphs of Linear Equations**

Use the equation $y = -\frac{3}{2}x + 6$ for all parts of this problem. Label all plotted points.

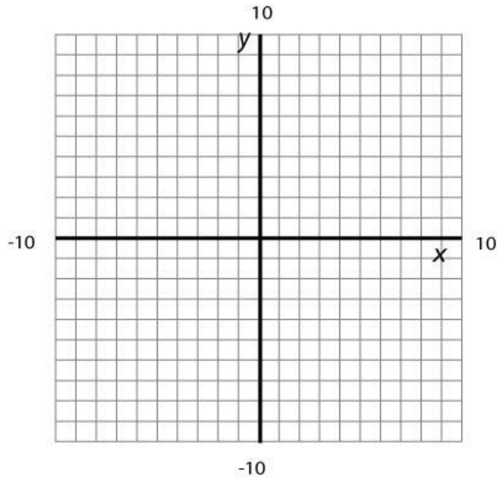
a) Use the INTERCEPTS to draw the graph of the line. Show your work to find these points. PLOT and LABEL the intercepts on the graph then connect them to draw your line.



Vertical Intercept (y-int): (_____ , _____)

Horizontal Intercept (x-int) : (_____ , _____)

b) Use the SLOPE to graph the line. Identify at least two additional points on the line (not the intercepts), and label them on your graph.



Slope = ___________

Two additional ordered pairs

(_____ , _____) (_____ , _____)

NOTICE: Your graphs for parts a) and b) should look exactly the same.

Section 3.3 – Horizontal and Vertical Lines

Vertical Lines

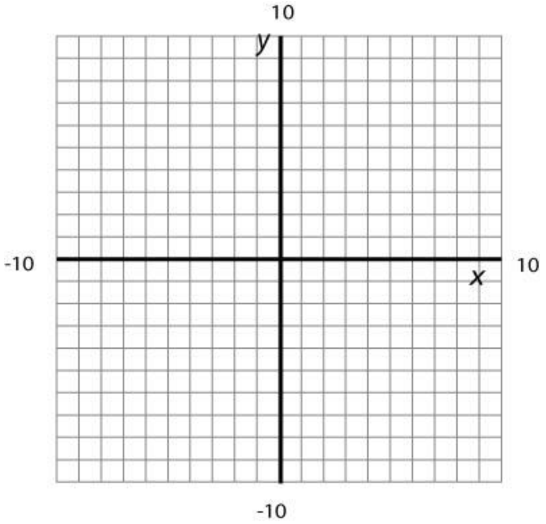
- Equation: $x = a$
- Horizontal Intercept (x-int): $(a, 0)$
- Vertical Intercept (y-int): none
- Slope: m is undefined
- Domain: $\{x = a\}$
- Range: $(-\infty, \infty)$

Horizontal Lines

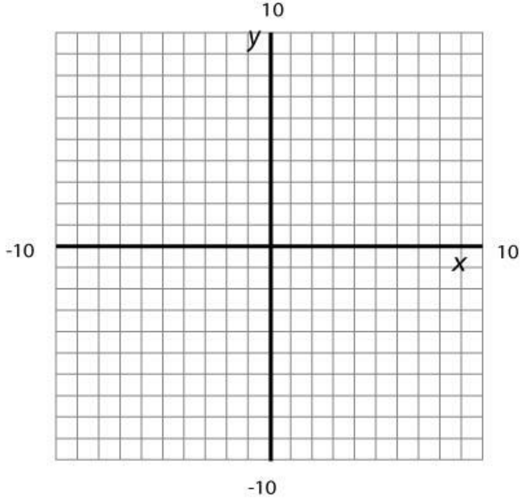
- Equation: $y = b, f(x) = b$
- Horizontal Intercept (x-int): none
- Vertical Intercept (y-int): $(0, b)$
- Slope: $m = 0$
- Domain: $(-\infty, \infty)$
- Range: $\{x = a\}$

Problem 12	MEDIA EXAMPLE – Graphing Horizontal and Vertical Lines

a) Use the grid below to graph the equation $y = -2$. Identify the slope and intercepts.



b) Use the grid below to graph the equation $x = 5$. Identify the slope and intercepts.



Problem 13	**YOU TRY – HORIZONAL AND VERTICAL LINES**

a) Given the ordered pair (2, –3)

i) Sketch the graph of the vertical line through this point.

ii) Write the equation of the vertical line through this point. Use function notation if possible.

iii) Identify the slope of the line: __________

iv) What is the Vertical Intercept (y-int)? ____________

v) What is the Horizontal Intercept (x-int)? ____________

b) Given the ordered pair (2, –3)

i) Sketch the graph of the horizontal line through this point.

ii) Write the equation of the horizontal line through this point. Use function notation if possible.

iii) Identify the slope of the line: __________

iv) What is the Vertical Intercept (y-int)? ____________

v) What is the Horizontal Intercept (x-int)? ____________

Section 3.4 – Writing the Equation of a Line

Writing Equations of Lines

Critical to a thorough understanding of linear equations and functions is the ability to write the equation of a line given different pieces of information. The following process will work for almost every situation you are presented with and will be illustrated several times in the media problems to follow.

Step 1: Determine the value of the slope, m.

Step 2: Determine the coordinates of one ordered pair.

Step 3: Plug the values for the ordered pair, and the value for the slope, into $y = mx + b$

Step 4: Solve for b

Step 5: Use the values for m and b to write the resulting equation in $y = mx + b$ form.

Step 6: When appropriate, rewrite the equation in function notation: $f(x) = mx + b$.

Problem 14	**MEDIA EXAMPLE – Writing Equations of Lines**

For each of the following, find the equation of the line that meets the following criteria:

a) Slope $m = –4$ passing through the point (0, 3).

b) Passing through the points (0, –2) and (1, 5)

c) Passing through the points (–2, –3) and (4, –9)

d) Parallel to $y = 3x - 7$ and passing through (2, –5)

e) Horizontal line passing through (–3, 5).

f) Vertical line passing through (–3, 5).

Problem 15	**WORKED EXAMPLE – Writing Equations of Lines**

Write an equation of the line to satisfy each set of conditions.

a) A line that contains the points (–3, 5) and (0, 1)

Slope: Use the ordered pairs (–3, 5) and (0, 1) to compute slope.

$$m = \frac{1-5}{0-(-3)} = \frac{-4}{3} = -\frac{4}{3}$$

Vertical Intercept (y-int): The Vertical Intercept (y-int) (0, 1) is given in the problem, so $b = 1$.

Equation: Plug $m = -\frac{4}{3}$ and $b = 1$ into $y = mx + b$

$$y = -\frac{4}{3}x + 1$$

$$f(x) = -\frac{4}{3}x + 1$$

---

b) Line contains points (–4, –3) and (2, 6)

Slope: Use the ordered pairs (–4, –3) and (2, 6) to compute slope.

$$\begin{aligned} m &= \frac{6-(-3)}{2-(-4)} \\ &= \frac{9}{6} \\ &= \frac{3}{2} \end{aligned}$$

Vertical Intercept (y-int): Because neither of the given ordered pairs is the Vertical Intercept (y-int), b must be computed. Pick one of the given ordered pairs. Plug m and that ordered pair into $y = mx+b$. Solve for b.

Using (–4, –3)		Using (2, 6)
$-3 = \frac{3}{2}(-4) + b$	or	$6 = \frac{3}{2}(2) + b$
$-3 = -6 + b$		$6 = 3 + b$
$3 = b$		$3 = b$

Equation: Plug $m = \frac{3}{2}$ and $b = 3$ into $y = mx + b$

$$y = \frac{3}{2}x + 3$$

$$f(x) = \frac{3}{2}x + 3$$

Problem 16	**YOU TRY – Writing Equations of Lines**

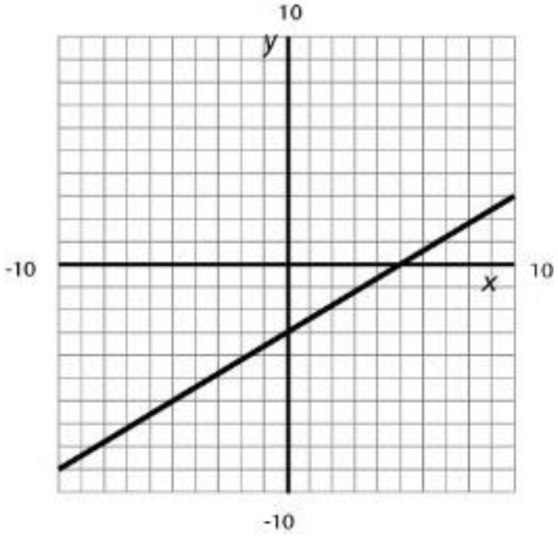
a) Find the equation of the line passing through the points (1,4) and (3, –2) and write your answer in the form $f(x) = mx + b$. Show complete work in this space.

b) What is the Vertical Intercept (y-int) for this equation? Show work or explain your result.

c) What is the Horizontal Intercept (x-int) for this equation? Show complete work to find this. Write the answer as a fraction and a decimal rounded to two decimal places.

Problem 17	**WORKED EXAMPLE – Writing Linear Equations from Graphs**

A line has the following graph:



Slope: Identify two ordered pairs from the graph and use them to determine the slope.

(5, 0) and (0, –3)

$$m = \frac{-3-0}{0-5} = \frac{-3}{-5} = \frac{3}{5}$$

Vertical Intercept (y-int): The Vertical Intercept (y-int) is given as one of the points chosen.

Ordered pair is (0, –3). Therefore $b = -3$.

Equation: Plug m and b into $y = mx + b$

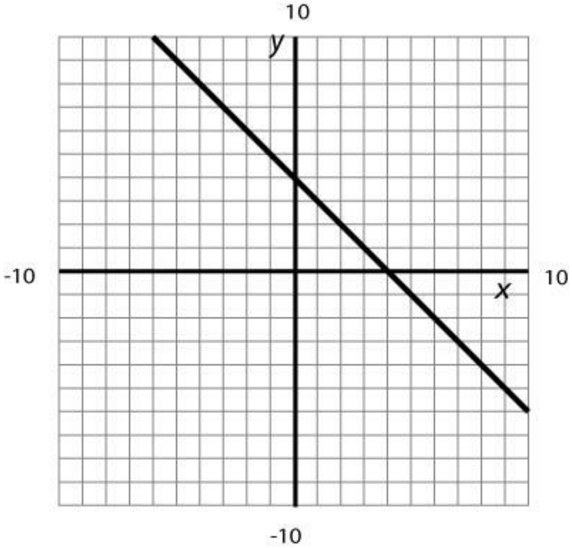
$m = \frac{3}{5}, b = -3$

$y = \frac{3}{5}x - 3$

$f(x) = \frac{3}{5}x - 3$

Problem 18	**YOU TRY – Writing Linear Equations from Graphs**

Use the given graph of the function *f* below to help answer the questions below. Assume the line intersects grid corners at integer (not decimal) values.



a) Is the line above increasing, decreasing, or constant?

b) What is the Vertical Intercept (y-int)? *Also, plot and label the Vertical Intercept (y-int) on the graph.*

c) What is the horizontal intercept? *Also, plot and label the Horizontal Intercept (x-int) on the graph.*

d) What is the slope? Show your work.

e) What is the equation of the line? Show complete work. Your answer must be written in function notation.

Problem 19	MEDIA EXAMPLE – Applications of Linear Functions

A candy company has a machine that produces candy canes. The number of candy canes produced depends on the amount of time the machine has been operating. The machine produces 160 candy canes in five minutes. In twenty minutes, the machine can produce 640 candy canes.

a) Determine a linear equation to model this situation. Clearly indicate what each variable represents.

b) Determine the Vertical Intercept (y-int) of this linear equation. Write it as an ordered pair and interpret its practical meaning.

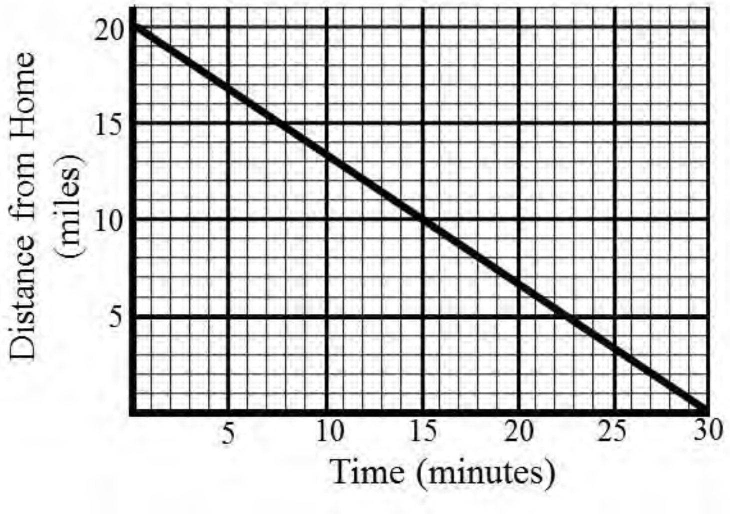
c) Determine the Horizontal Intercept (x-int) of this linear equation. Write it as an ordered pair and interpret its practical meaning.

d) How many candy canes will this machine produce in 1 minute?

e) How many candy canes will this machine produce in 1 hour?

Problem 20	**YOU TRY – Applications of Linear Functions**

The graph below shows a person's distance from home as a function of time.



a) Identify the Vertical Intercept (y-int). Write it as an ordered pair and interpret its practical meaning.

b) Identify the Horizontal Intercept (t-int). Write it as an ordered pair and interpret its practical meaning.

c) Determine a linear equation to model this situation. Indicate what each variable represents.

d) How far has this person traveled in one minute?

Lesson 3 Practice Problems

Round to two decimal places unless stated otherwise.

Section 3.1: Linear Equations and Functions

1. Find the slope of the line that passes through the given points. Then determine if the line is increasing, decreasing or constant.

	Points	Slope	Sign of Slope (+, −, 0)	Increasing, Decreasing or constant?
a)	(3, 2) and (6, 8)	$m = \frac{8-2}{6-3} = \frac{6}{3} = 2$	*Positive*	*Increasing*
b)	(−2, 6) and (−6, −2)			
c)	(3, −5) and (7, 7)			
d)	(−1, −5) and (4, 7)			
e)	(−3, 12) and (5, −1)			
f)	$\left(\frac{3}{2}, 2\right)$ and $\left(-\frac{5}{2}, 2\right)$			
g)	$\left(-\frac{3}{4}, \frac{2}{7}\right)$ and $\left(-\frac{1}{4}, \frac{4}{7}\right)$			

2. Complete the table below. If the equation is not in $y = mx + b$ form, show the steps required to convert it to that form. Also show the work required to calculate the Horizontal Intercept (x-int). Write all intercepts as ordered pairs. Write the answer as a fraction and a decimal rounded to two decimal places.

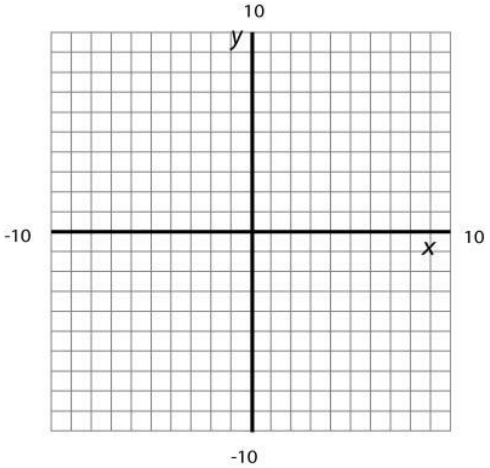
Equation	$y = mx + b$ form	Slope	Vertical Intercept (y-int)	Horizontal Intercept (x-int)
a) $y = -4x - 8$				
b) $y = 3 - 4x$				
c) $y = \frac{1}{3}x - 2$				
d) $-4x - y = 2$				
e) $-6x + 3y = 9$				
f) $y = 2x$				
g) $y = 4$				
h) $x = -3$				

Section 3.2: Graphs of Linear Functions

3. For the given linear functions, complete the table of values. Plot the ordered pairs, and graph the line.

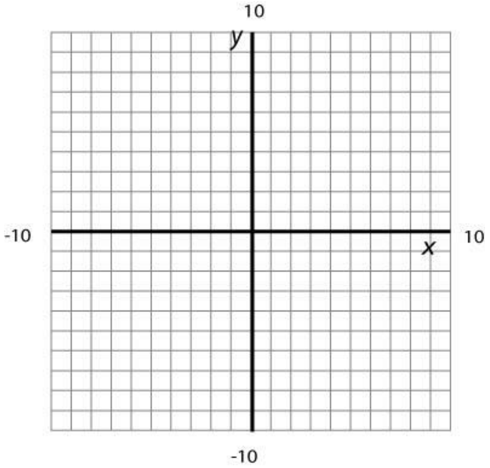
a) $y = 3x - 2$

x	$y = 3x - 2$	Ordered Pair
−3	$y = 3(-3) - 2 = -11$	$(-3, -11)$
−2		
−1		
0		
1		
2		
3		



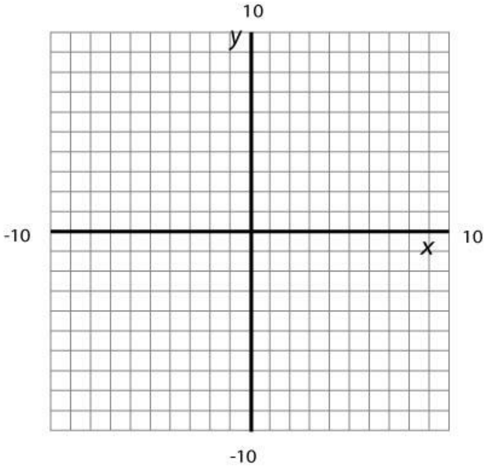
b) $y = -2x + 4$

x	$y = -2x + 4$	Ordered Pair
−3		
−2		
−1		
0		
1		
2		
3		



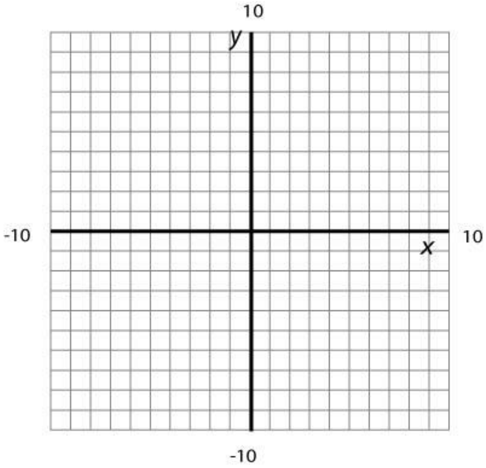
c) $y = -\frac{3}{2}x + 1$

x	$y = -\frac{3}{2}x + 1$	Ordered Pair
−4		
−2		
0		
2		
4		



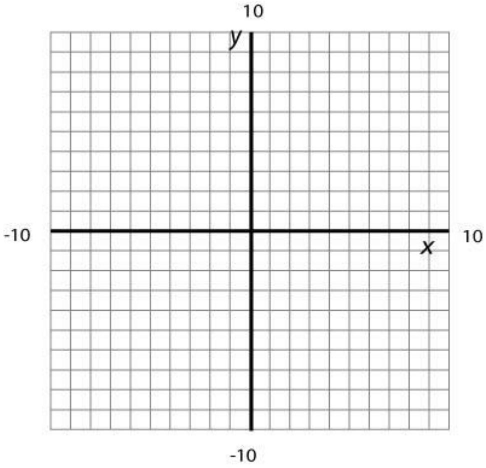
d) $y = \frac{2}{5}x - 3$

x	$y = \frac{2}{5}x - 3$	Ordered Pair
−10		
−5		
0		
5		
10		

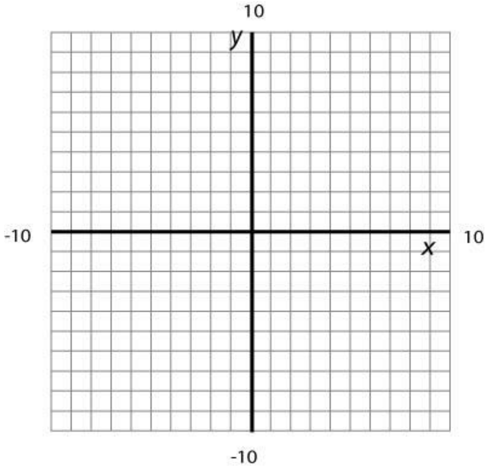


e) $y = -x$

x	$y = -x$	Ordered Pair
−3		
−2		
−1		
0		
1		
2		
3		



4. Use the intercepts to draw the graph of the equation $y = 3x - 1$. Show your work to find these points. PLOT and LABEL the intercepts on the graph then connect them to draw the line.

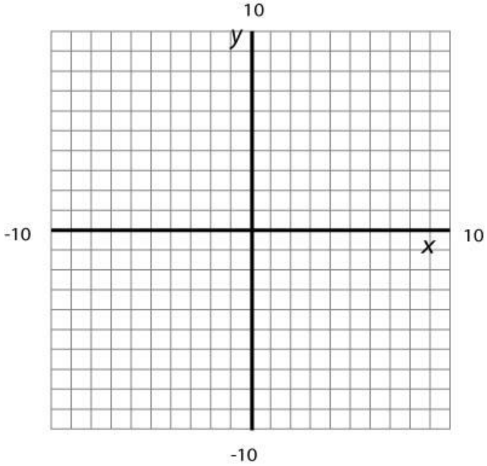


Vertical Intercept (y-int):
(_____ , _____)

Write the answer as a fraction and a decimal rounded to two decimal places.

Horizontal Intercept (x-int) :
(_____ , _____)=(_____ , _____)

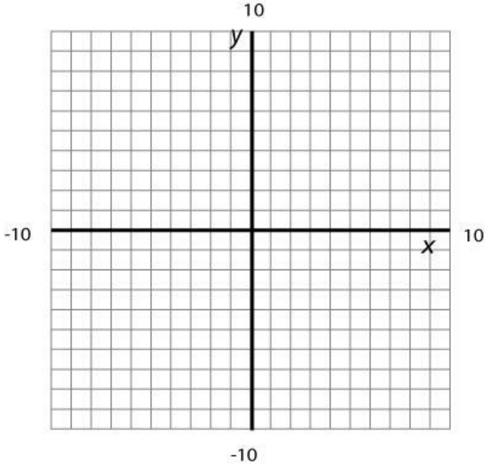
5. Draw the graph of the equation $y = -x + 2$ Show your work to find these points. PLOT and LABEL the intercepts on the graph then connect them to draw the line.



Vertical Intercept (y-int):
(_____ , _____)

Horizontal Intercept (x-int):
(_____ , _____)

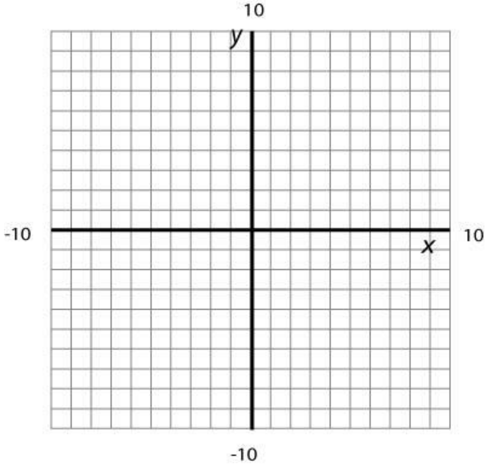
6. Draw the graph of the equation $y = -\frac{1}{3}x + 3$ Show your work to find these points. PLOT and LABEL the intercepts on the graph then connect them to draw the line.



Vertical Intercept (y-int):
(_____ , _____)

Horizontal Intercept (x-int):
(_____ , _____)

7. Draw the graph of the equation $2x - 3y = 12$ Show your work to find these points. PLOT and LABEL the intercepts on the graph then connect them to draw the line.

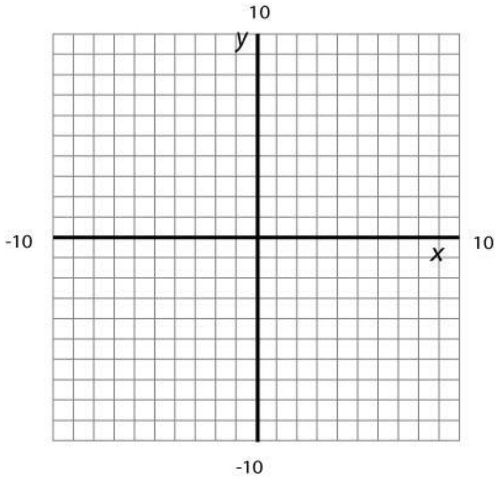


Vertical Intercept (y-int):
(_____ , _____)

Horizontal Intercept (x-int) :
(_____ , _____)

8. Use the SLOPE to graph the line. Identify at least two additional points on the line (not the intercepts), and label them on your graph.

a) A line has a slope of 4 and contains the point $(-3,0)$.



Plot the point on the graph to the right and use the *slope* to find at least two other points to graph the line.

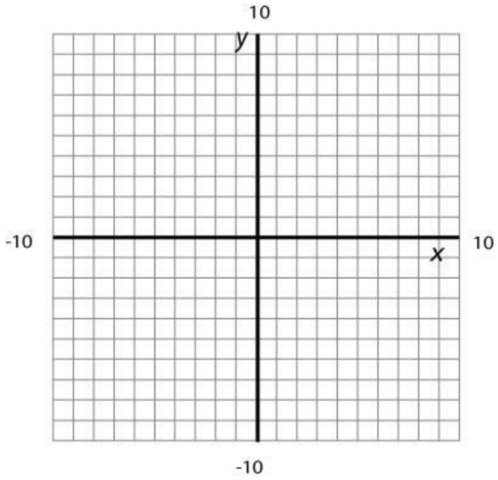
List the two additional ordered pairs you found below.

(_____ , _____) (_____ , _____)

Use the *slope* to complete the table below.

x	y
−3	0
−5	
−1	

b) A line has a slope of -2 and contains the point $(-1,5)$.



Plot the point on the graph to the right and use the *slope* to find at least two other points to graph the line.

List the two additional ordered pairs you found below.

(_____ , _____) (_____ , _____)

Use the *slope* to complete the table below.

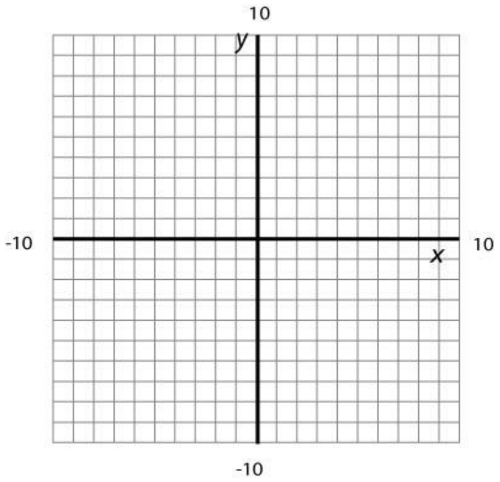
x	y
−3	
0	
6	

c) A line has a slope of $-\frac{1}{3}$ and contains the point (2,−4).

Plot the point on the graph to the right and use the *slope* to find at least two other points to graph the line.

List the two additional ordered pairs you found below.

(_____ , _____)   (_____ , _____)



Use the *slope* to complete the table below.

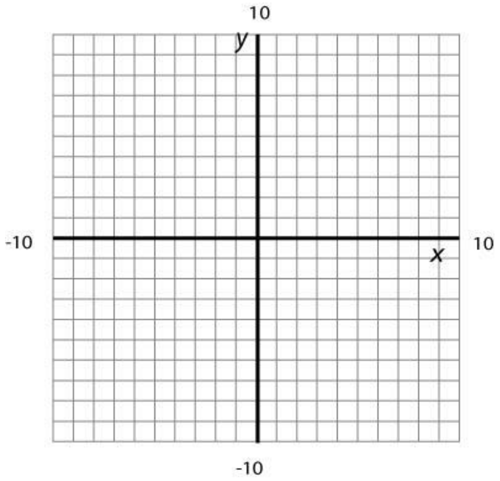
x	y
2	−4
8	
−10	

d) A line has a slope of $\frac{1}{2}$ and contains the point (−3, 2).

Plot the point on the graph to the right and use the *slope* to find at least two other points to graph the line.

List the two additional ordered pairs you found below.

(_____ , _____)   (_____ , _____)



Use the *slope* to complete the table below.

x	y
−3	2
−7	
7	

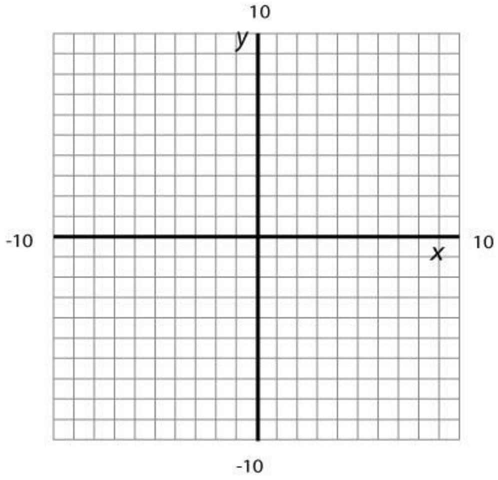
Section 3.3: Horizontal and Vertical Lines

9. Complete the table.

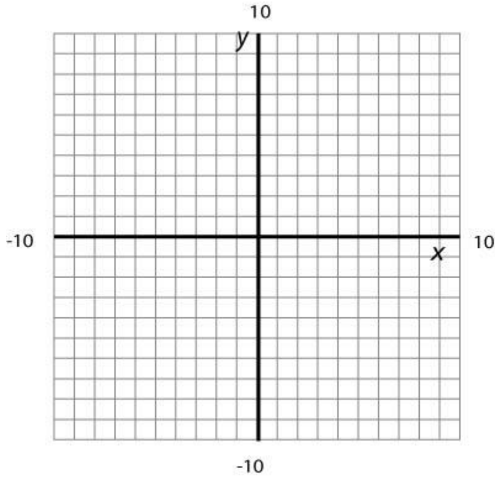
Equation	Slope	Horizontal Intercept (x-int)	Vertical Intercept (y-int)
a) $y = 5$	$m = 0$	*does not exist*	(0, 5)
b) $y = 3$			
c) $x = 3$			
d) $y = -2$			
e) $x = -4$			
f) $y = 0$			
g) $x = 0$			

10. Graph each of the following equations. Plot and label any intercepts.

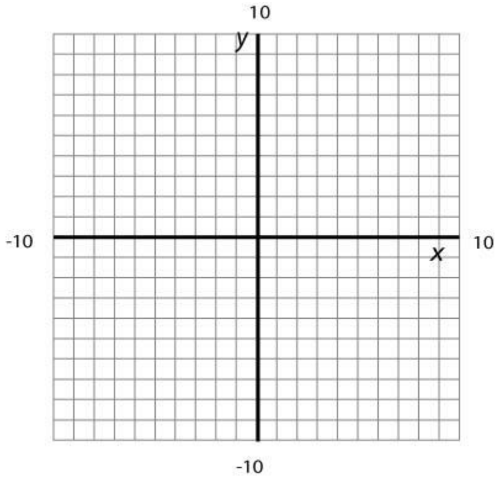
a) $y = 3$



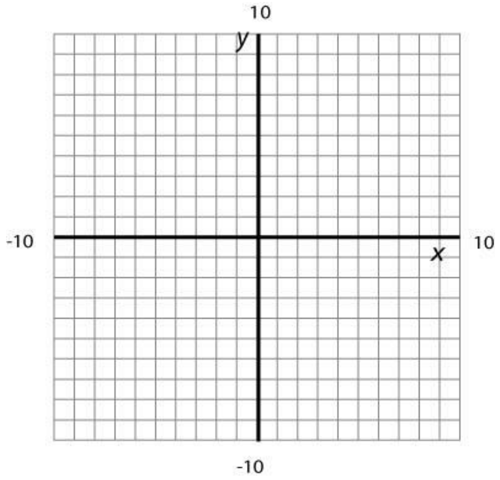
b) $x = 3$



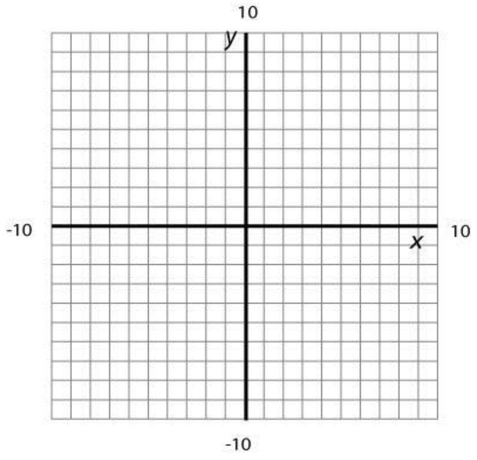
c) $y = -2$



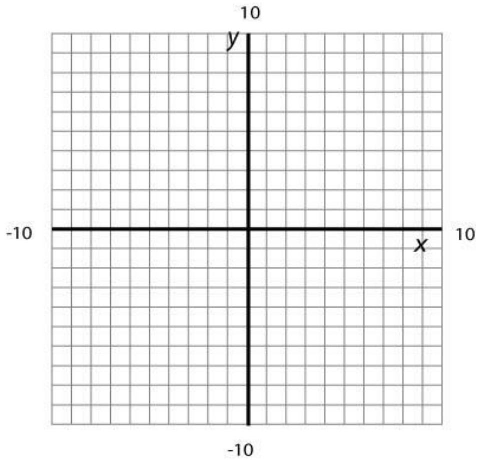
d) $x = -4$



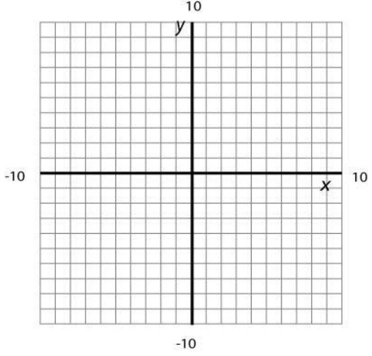
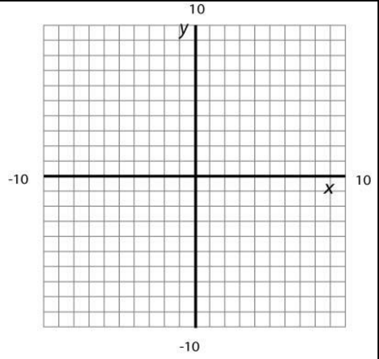
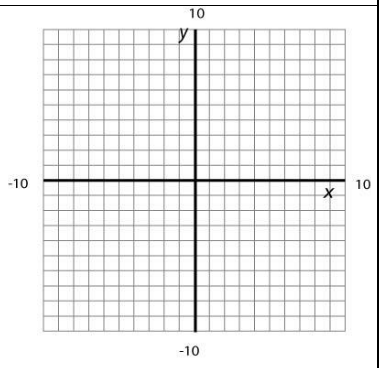
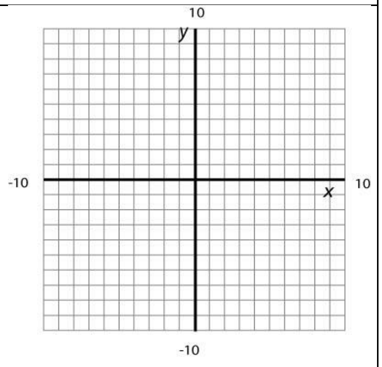
e) $y = 0$



f) $x = 0$



11. Use the given information to determine the equation of the line and to graph the line.

	Given Information	Equation	Graph
a)	A horizontal line that passes through the point $(-2, 3)$		
b)	A vertical line that passes through the point $(5, -2)$		
c)	A horizontal line that passes through the point $(8, 3)$		
d)	A vertical line that passes through the point $(-4, -7)$		

Section 3.4: Writing the Equation of a Line

12. For each of the following, find the equation of the line that meets the following criteria.

	Slope	Point	Equation of Line
a)	$m = 2$	$(0, -3)$	
b)	$m = -4$	$(0, \frac{2}{3})$	
c)	$m = \frac{3}{8}$	$(0, -5)$	
d)	$m = -2.37$	$(0, 6.35)$	

13. For each of the following, find the equation of the line that meets the following criteria.

	Slope	Point	Find Vertical Intercept (y-int)	Equation of Line
a)	$m = 2$	$(2, -3)$		
b)	$m = -4$	$(3,4)$		
c)	$m = \frac{5}{16}$	$(-8, -5)$		
d)	$m = -1.4$	$(2, 2.34)$		

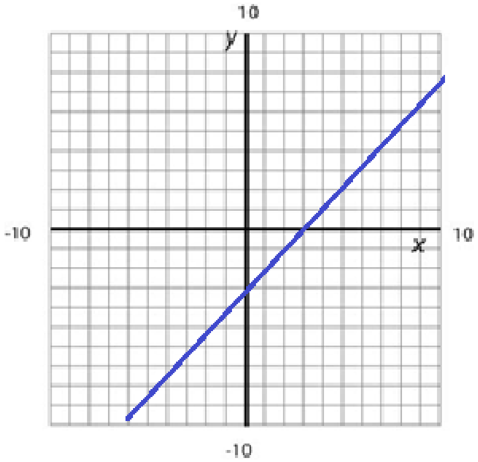
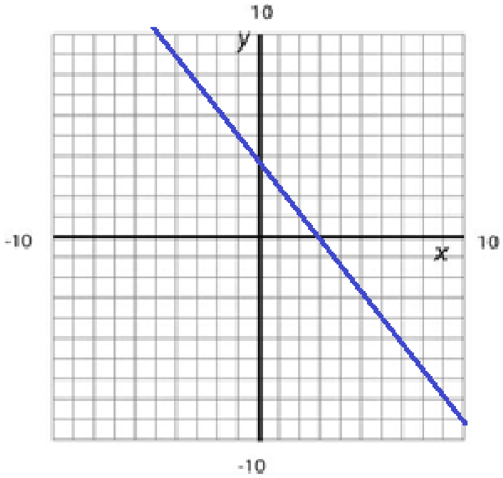
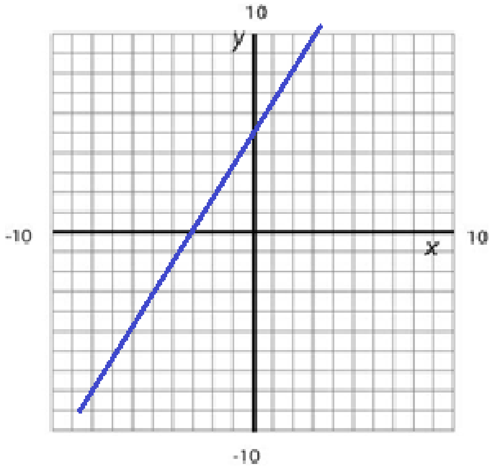
14. For each of the following, find the equation of the line that meets the following criteria.

	Two Points	Find Slope	Find Vertical Intercept (y-int)	Equation of Line
a)	(2, −3), (4, 7)			
b)	(−3, 6), (3, −12)			
c)	(5, −5), (−1, 3)			
d)	(2, 4.2), (6, 9.4)			

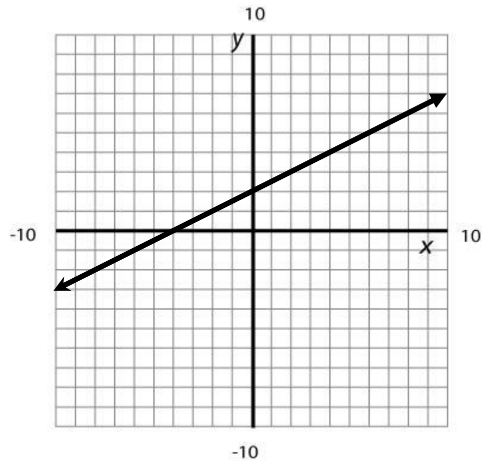
15. Determine the equation of the line that is parallel to the given line and passes through the point.

	Equation of given line	Point on parallel line	Slope of parallel line	Vertical Intercept (y-int) of parallel line	Equation of Parallel line
a)	$y = 2x - 4$	(2, −3)			
b)	y $= -3x + 4$	(3,4)			
c)	$y = \frac{3}{2}x + 2$	(−8, −5)			

16. Determine the equation of the line that corresponds to the given graph.

Graph	Equation
a) 	
b) 	
c) 	

17. Use the given graph to help answer the questions below.



a) Is the line above increasing, decreasing, or constant?

b) What is the Vertical Intercept (y-int)? Also, plot and label the point on the graph.

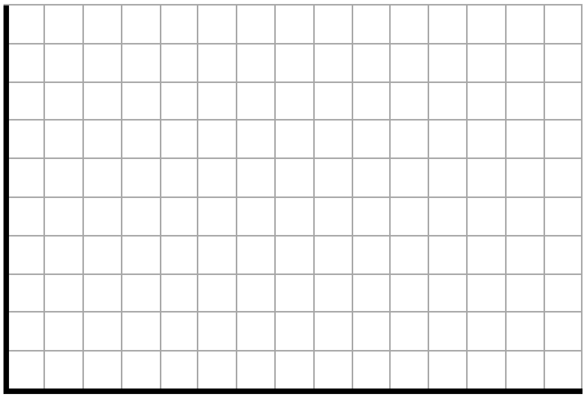
c) What is the Horizontal Intercept (x-int)? Also, plot and label the point on the graph.

d) What is the slope? Show your work.

e) What is the equation of the line in $y = mx + b$ form?

18. Find the equation of the line for the following problem. Clearly indicate what your variables represent. Graph the results.

Cora decided to go on a diet. On the day she started, she weighed 200 pounds. For the next 8 weeks, she consistently lost 2 pounds a week. At the end of 8 weeks, she decided to make a graph showing her progress.



19. Mark needed 200 pounds of roofing nails for his project. He poured one cup filled with nails into a bucket and found that it weighed 2.3 pounds. He then poured 4 more cups of nails into the bucket and found that it weighed 9.5 pounds. He figured if he used the points **(1, 2.3) and (5, 9.5)** he could figure out a formula (i.e. equation) and calculate how many cups he would need.

a) Find the equation of the line for this problem. Clearly indicate what your variables represent.

b) How many cups of roofing nails does Mark need for his project?

c) Challenge question. The formula you found above doesn't go through the origin. Shouldn't 0 cups of nails weigh 0 pounds? Can you figure out why 0 cups of nails actually weighs MORE than 0 pounds in Mark's equation?

Lesson 3 Assessment

1. Determine the equation of the line between the points (4, 3) and (12, –3). Write your answer in slope-intercept form $f(x) = mx + b$.

2. Write the equation of the line parallel to $y = 5x - 7$ and passing through the point (1,2).

3. The function $P(n) = 455n - 1820$ represents a computer manufacturer's profit when n computers are sold.

a) Identify the Vertical Intercept (y-int). **Write it as an ordered pair and interpret its practical meaning in a complete sentence.**

Ordered Pair:_________________

b) Determine the Horizontal Intercept (x-int). **Write it as an ordered pair and interpret its practical meaning in a complete sentence.**

Ordered Pair:_________________

4. Determine the equation of the vertical line passing through the point (4, 7). _____________

5. The x-axis is a line. Write the equation of this line. ___________

6. Fill in the table below. Intercepts <u>must</u> be written as ordered pairs. Write "DNE" if your answer does not exist (is undefined). Write the answer as a fraction and a decimal rounded to two decimal places.

I = Increasing D = Decreasing C = Constant (Horizontal) V = Vertical

Equation	Slope	Vertical Intercept	Horizontal Intercept	Behavior I, D, C, V
$y = 2x - 16$				
$f(x) = 8 - 3x$				
$y = -2$				
$y = x$				
	–5	(0, 20)		
		(0, –9)	(–3, 0)	
$x = 6$				

Lesson 4 – Linear Functions and Applications

In this lesson, we take a close look at Linear Functions and how real world situations can be modeled using Linear Functions. We study the relationship between Average Rate of Change and Slope and how to interpret these characteristics. We also learn how to create Linear Models for data sets using Linear Regression.

Lesson Topics:

Section 4.1: Review of Linear Functions

Section 4.2: Average Rate of Change

- Average Rate of Change as slope
- Interpret the Average Rate of Change
- Use the Average Rate of Change to determine if a function is Linear

Section 4.3: Scatterplots on the Graphing Calculator

Section 4.4: Linear Regression

- Using your graphing calculator to generate a Linear Regression equation
- Using Linear Regression to solve application problems

Section 4.5: Multiple Ways to Determine the Equation of a Line

Mini-Lesson 4

Section 4.1 – Review of Linear Functions

This lesson will combine the concepts of FUNCTIONS and LINEAR EQUATIONS. To write a linear equation as a LINEAR FUNCTION, replace the variable y using FUNCTION NOTATION. For example, in the following linear equation, we replace the variable y with $f(x)$:

$$y = mx + b$$

$$f(x) = mx + b$$

Important Things to Remember about the LINEAR FUNCTION $f(x) = mx + b$

- x represents the INPUT quantity.
- $f(x)$ represents the OUTPUT quantity.
- The graph of f is a straight line with slope, m, and Vertical Intercept (y-int) $(0, b)$.
- Given any two points (x_1, y_1) and (x_2, y_2) on a line,

$$m = \frac{\text{Change in Output}}{\text{Change in Input}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x}$$

- If $m > 0$, the graph INCREASES from left to right,
  If $m < 0$, the graph DECREASES from left to right,
  If $m = 0$, then $f(x)$ is a CONSTANT function, and the graph is a horizontal line.
- The DOMAIN of a Linear Function is generally ALL REAL NUMBERS unless a context or situation is applied in which case we interpret the PRACTICAL DOMAIN in that context or situation.
- One way to identify the Vertical Intercept (y-int) is to evaluate $f(0)$. In other words, substitute 0 for input (x) and determine the resulting output.
- To find the Horizontal Intercept (x-int), solve the equation $f(x) = 0$ for x. In other words, set
  $mx + b = 0$ and solve for the value of x. Then $(x, 0)$ is your Horizontal Intercept (x-int).

Problem 1	**YOU TRY – Review of Linear Functions**

The function $E(t) = 3861 - 77.2t$ gives the surface elevation (in feet above sea level) of Lake Powell t years after 1999.

a) Identify the Vertical Intercept (y-int) of this linear function and write a sentence explaining its meaning in this situation.

b) Determine the surface elevation of Lake Powell in the year 2001. Show your work, and write your answer in a complete sentence.

c) Determine E(4), and write a sentence explaining the meaning of your answer.

d) Is the surface elevation of Lake Powell increasing or decreasing? How do you know?

e) This function accurately models the surface elevation of Lake Powell from 1999 to 2005. Determine the practical range of this linear function.

Section 4.2 – Average Rate of Change

Average rate of change of a function over a specified interval is the ratio:

$$\text{Average Rate of Change} = \frac{\text{Change in Output}}{\text{Change in Input}} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x}$$

Units for the Average Rate of Change are always $\frac{\text{output units}}{\text{input unit}}$,

which can be interpreted as "output units *per* input unit"

Problem 2 | **MEDIA EXAMPLE – Average Rate of Change**

The function $E(t) = 3861 - 77.2t$ gives the surface elevation of Lake Powell t years after 1999. Use this function and your graphing calculator to complete the table below.

t, years since 1999	$E(t)$, Surface Elevation of Lake Powell (in feet above sea level)
0	
1	
2	
3	
5	
6	

a) Determine the Average Rate of Change of the surface elevation between 1999 and 2000.

b) Determine the Average Rate of Change of the surface elevation between 2000 and 2004.

c) Determine the Average Rate of Change of the surface elevation between 2001 and 2005.

d) What do you notice about the Average Rates of Change for the function $E(t)$?

e) On the grid below, draw a GOOD graph of E(t) with all appropriate labels.

Because the Average Rate of Change is constant for these data,
we say that a LINEAR FUNCTION models these data best.

Does AVERAGE RATE OF CHANGE look familiar? It should! Another word for "average rate of change" is SLOPE of a line. Given any two points (x_1, y_1) and (x_2, y_2) on a line, the slope is determined by computing the following ratio:

$$m = \frac{\text{Change in Output}}{\text{Change in Input}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta \text{y}}{\Delta \text{x}} = \frac{\text{Rise}}{\text{Run}}$$

Therefore, AVERAGE RATE OF CHANGE = SLOPE of a line over a given interval.

Average Rate of Change

- Given any two points (x_1, y_1) and (x_2, y_2), the average rate of change between the points on the interval x_1 to x_2 is determined by computing the following ratio:

$$\text{Average Rate of Change} = \frac{\text{Change in Output}}{\text{Change in Input}} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x}$$

- If the function is LINEAR, then the average rate of change will be *the same* between any pair of points.
- If the function is LINEAR, then the average rate of change is the SLOPE of the linear function.

Problem 3 MEDIA EXAMPLE – Is the Function Linear?

For each of the following, determine if the function is linear. If it is linear, give the slope.

a)

x	-4	-1	2	8	12	23	42
y	-110	-74	-38	34	82	214	442

b)

x	-1	2	3	5	8	10	11
y	5	-1	1	11	41	71	89

c)

x	-4	-1	2	3	5	8	9
y	42	27	12	7	-3	-18	-23

Problem 4	**YOU TRY – Is the Function Linear?**

For each of the following, determine if the function is linear. If it is linear, give the slope.

a)

x	-5	-2	1	4	6	8	11
$f(x)$	491	347	203	59	-37	-133	-277

b)

n	-8	-5	-2	0	3	4	9
$A(n)$	6.9	7.5	8.1	8.5	9.1	9.3	10.3

c)

t	-3	0	1	5	8	11	15
$g(t)$	-4	-2	0	2	4	6	8

Problem 5	**MEDIA EXAMPLE – Average Rate of Change and Linear Functions**

The data below represent your annual salary for the first four years of your current job.

Time, *t*, in years	0	1	2	3	4
Salary, S, in thousands of dollars	20.1	20.6	21.1	21.6	22.1

a) Identify the Vertical Intercept (y-int). Write it as an ordered pair and interpret its meaning in a complete sentence.

b) Determine the average rate of change during this 4-year time period. Write a sentence explaining the meaning of the average rate of change in this situation. Be sure to include units.

c) Verify that the data represent a linear function by computing the average rate of change between two additional pairs of points.

d) Write the linear function model for the data. Use the indicated variables and proper function notation.

Problem 6	**YOU TRY – Average Rate of Change**

The data below show a person's body weight during a 5-week diet program.

Time, *t*, in weeks	0	1	2	3	4	5
Weight, W, in pounds	196	192	193	190	190	186

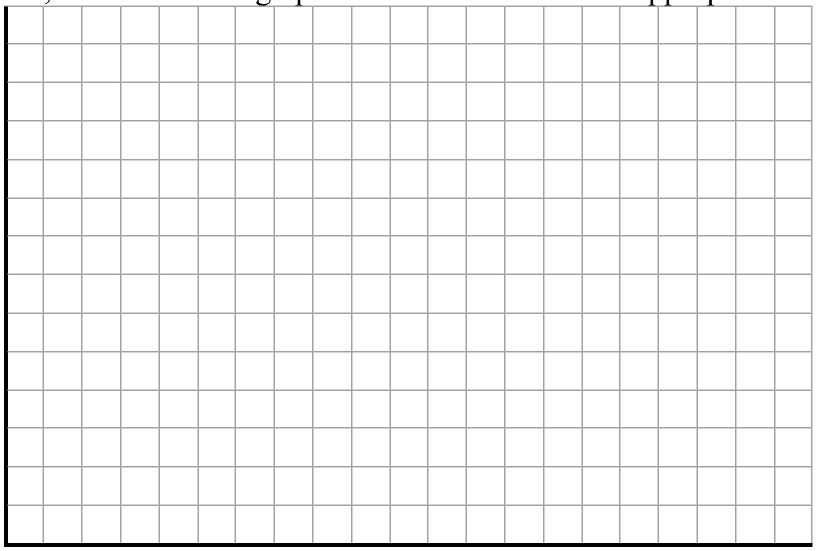
a) Identify the Vertical Intercept (y-int). Write it as an ordered pair and write a sentence explaining its meaning in this situation.

b) Compute the average rate of change for the 5-week period. Be sure to include units.

c) Write a sentence explaining the meaning of your answer in part b) in the given situation.

d) Do the data points in the table define a perfectly linear function? Why or why not?

e) On the grid below, draw a GOOD graph of this data set with all appropriate labels.



Section 4.3 – Scatterplots on the Graphing Calculator

Consider the data set from the previous problem:

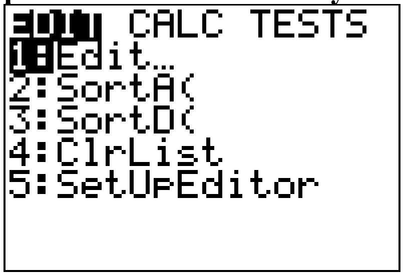
Time, *t*, in weeks	0	1	2	3	4	5
Weight, W, in pounds	196	192	193	190	190	186

In the next example, you will see how your graphing calculator can be used to generate a scatterplot from a given data set, much like the one you drew by hand in the previous problem.

Problem 7	**MEDIA EXAMPLE – Scatterplots on Your Graphing Calculator**

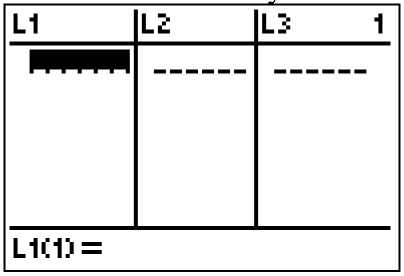
Watch the video and follow the steps on your calculator.

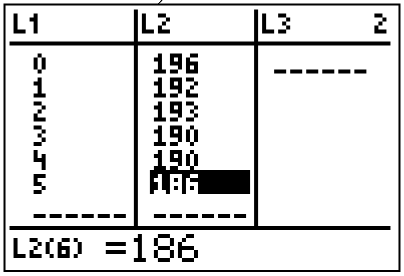
Step 1: Enter the data into your calculator



- Press STAT (Second Row of Keys)
- Press ENTER to access 1:Edit under EDIT menu
- *Note: Be sure all data columns are cleared. To do so, use your arrows to scroll up to L1 or L2 then click CLEAR then scroll down.* (DO NOT CLICK DELETE!)

Once your data columns are clear, enter the input data into L1 (press ENTER after each data value to get to the next row) then right arrow to L2 and enter the output data into L2. Your result should look like this when you are finished (for L1 and L2):

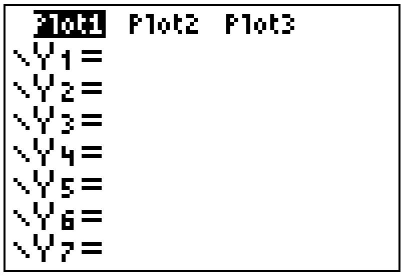




L1	L2	L3 2
0	196	------
1	192	
2	193	
3	190	
4	190	
5	186	
------	------	

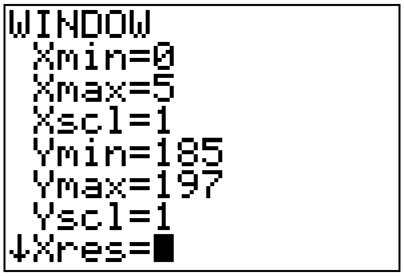
L2(6) =186

Step 2: Turn on your Stat Plot

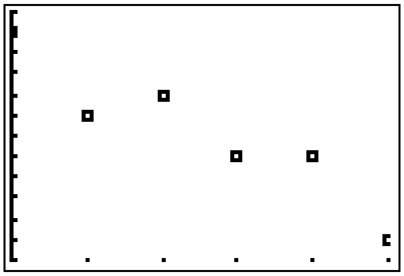


- Press Y=
- Use your arrow keys to scroll up to Plot1
- Press ENTER
- Scroll down and Plot1 should be highlighted as at left
- Clear out all entries below

Step 3: Graph the Data in an Appropriate Viewing Window



```
WINDOW
 Xmin=0
 Xmax=5
 Xscl=1
 Ymin=185
 Ymax=197
 Yscl=1
↓Xres=█
```



- Click the WINDOW key to set your viewing window
- Look at your data set, and determine the lowest and highest input values. In this data set, the lowest input value is 0 and the highest is 5. Set your xmin at (or just below) your lowest input value. Set your xmax at (or just above) your highest input value.
- Look at your data set, and determine the lowest and highest output values. In this data set, the lowest output value is 186 and the highest is 196. Set your ymin at (or just below) your lowest output value. Set your ymax at (or just above) your highest output value.
- Once your viewing window is set, click GRAPH. A graph of your data should appear in an appropriate window so that all data points are clearly visible.

**NOTE *If you ever accidentally DELETE a column, then go to STAT>5: SetUpEditor>ENTER. When you go back to STAT, your column should be restored.*

Problem 8	**YOU TRY – Scatterplots on Your Graphing Calculator**

Use your graphing calculator to create of scatterplot of the data set shown below. Be sure to use an appropriate viewing window.

X	4	12	18	26	44	57	71
Y	648	641	645	637	632	620	616

In the space below, sketch what you see on your calculator screen, and write down the viewing window you used.

Viewing Window:

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

Section 4.4 – Linear Regression

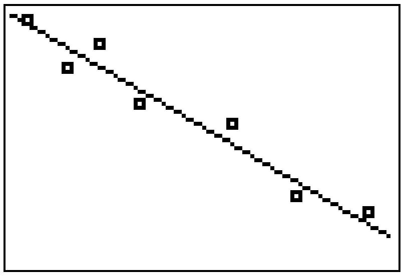
Just because data are not EXACTLY linear does not mean we cannot write an approximate linear model for the given data set.

In fact, most data in the real world are NOT exactly linear and all we can do is write models that are close to the given values. The process for writing Linear Models for data that are not perfectly linear is called LINEAR REGRESSION. If you take a statistics class, you will learn a lot more about this process. In this class, you will be introduced to the basics. This process is also called "FINDING THE LINE OF BEST FIT".

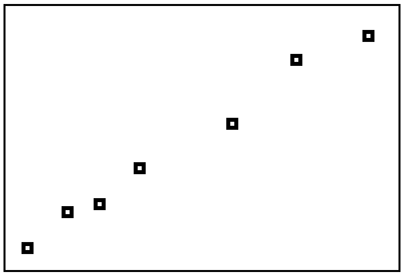
Problem 9 WORKED EXAMPLE – The Line of Best Fit

Below are the scatterplots of different sets of data. Notice that not all of them are *exactly* linear, but the data seem to follow a linear pattern. Using a ruler or straightedge, draw a straight line on each of the graphs that appears to "FIT" the data best. (Note that this line might not actually *touch* all of the data points.) The first one has been done for you.

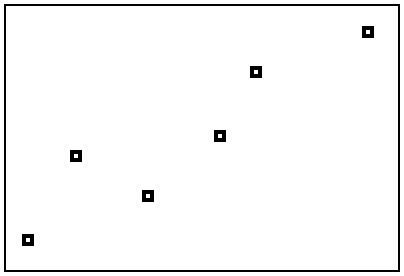
a)



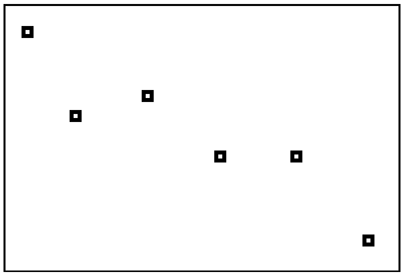
b)



c)



d)



To determine a linear equation that models the given data, we could do a variety of things. We could choose the first and last point and use those to write the equation. We could ignore the first point and just use two of the remaining points. Our calculator, however, will give us the best linear equation possible taking into account ALL the given data points. To find this equation, we use a process called LINEAR REGRESSION.

NOTE: Unless your data are exactly linear, the regression equation will not match all data points exactly. It is a model used to predict outcomes not provided in the data set.

Problem 10	**MEDIA EXAMPLE – Linear Regression**

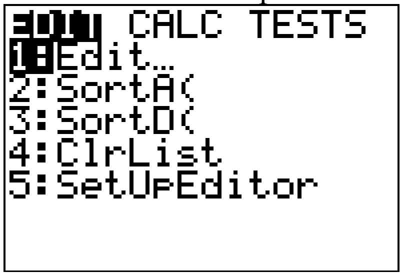
Watch the video and follow the steps on your calculator.

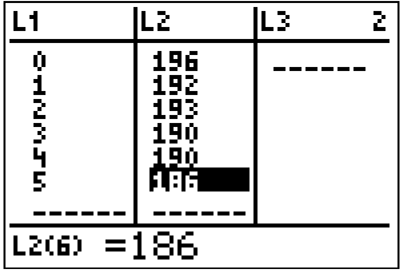
Consider the data set from the previous problem:

Time, *t*, in weeks	0	1	2	3	4	5
Weight, W, in pounds	196	192	193	190	190	186

Step 1: Enter the Data into your Graphing Calculator

Press STAT then select option 1:Edit under EDIT menu. Clear lists, then enter the values.

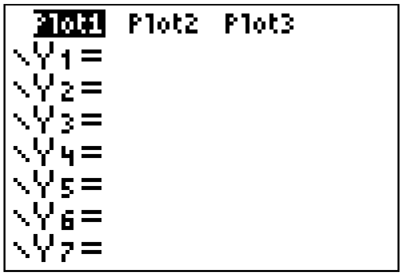


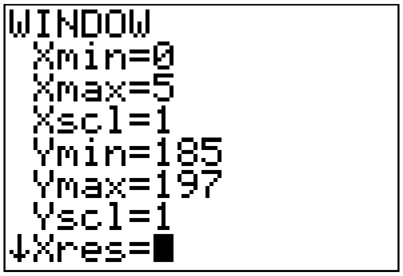


```
L1      L2      L3      2
0       196     ------
1       192
2       193
3       190
4       190
5
------  ------
L2(6) =186
```

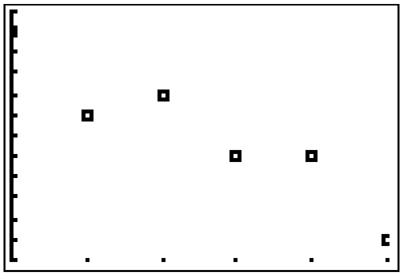
**NOTE *If you ever accidentally DELETE a column, then go to STAT>5: SetUpEditor>ENTER. When you go back to STAT, your column should be restored.*

Step 2: Turn on your Stat Plot and Graph the Data in an Appropriate Viewing Window
(Refer to previous example for help)

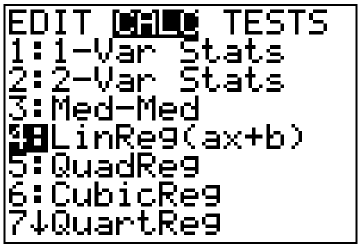




```
WINDOW
 Xmin=0
 Xmax=5
 Xscl=1
 Ymin=185
 Ymax=197
 Yscl=1
↓Xres=█
```

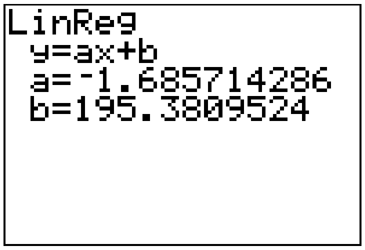


Step 3: Access the Linear Regression section of your calculator



- Press STAT
- Scroll to the right one place to CALC
- Scroll down to 4:LinReg(ax+b)
- Your screen should look as the one at left

Step 4: Determine the linear regression equation

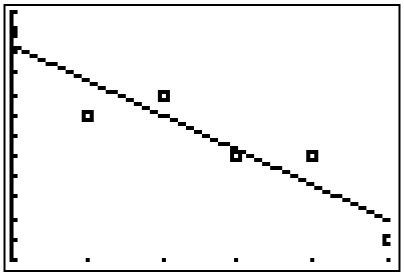


```
LinReg
 y=ax+b
 a=⁻1.685714286
 b=195.3809524
```

- Press ENTER repeatedly to get the screen shown to the left.
- The calculator computes values for slope (a) and y-intercept (b) in what is called the equation of best-fit for your data.
- Identify these values and round to the appropriate places. Let's say 2 decimals in this case.
  So, a = -1.69 and b = 195.38
- Now, replace the a and b in y = ax + b with the rounded values to write the actual equation:
  $y = -1.69x + 195.38$
- To write the equation in terms of initial variables, we would write $W = -1.69t + 195.38$
- In function notation, $W(t) = -1.69t + 195.38$

Once we have the equation figured out, it's nice to graph it on top of our data to see how things match up.

GRAPHING THE REGRESSION EQUATION ON TOP OF THE STAT PLOT



- Enter the Regression Equation with rounded values into Y=
- Press GRAPH
- You can see from the graph that the "best fit" line does not hit very many of the given data points. But, it will be the most accurate linear model for the overall data set.

IMPORTANT NOTE: When you are finished graphing your data, TURN OFF YOUR PLOT1. Otherwise, you will encounter an INVALID DIMENSION error when trying to graph other functions. To do this:

- Press Y=
- Use your arrow keys to scroll up to Plot1
- Press ENTER
- Scroll down and Plot1 should be UNhighlighted

Problem 11 | YOU TRY – Linear Regression

The function f is defined by the following table.

n	0	2	4	6	8	10	12
$f(n)$	23.76	24.78	25.93	26.24	26.93	27.04	27.93

a) Based on this table, determine $f(6)$. Write the specific ordered pair associated with this result.

b) Use your graphing calculator to determine the equation of the regression line for the given data. Round to three decimals as needed.

The regression equation in $y = ax + b$ form is: ______________________________

Rewrite the regression equation in function notation.

The regression equation in $g(n) = an + b$ form is: ______________________________

c) Use your graphing calculator to generate a scatterplot of the data *and* regression line on the same screen. You must use an appropriate viewing window. In the space below, draw what you see on your calculator screen, and write down the viewing window you used.

Xmin=____________

Xmax=____________

Ymin=____________

Ymax=___________

d) Using your REGRESSION EQUATION, determine $g(6)$. Write the specific ordered pair associated with this result.

e) Your answers for a) and d) should be different. Why is this the case? (refer to Problem 9 for help).

Problem 12	**YOU TRY – Linear Regression**

The following table gives the total number of live Christmas trees sold, in millions, in the United States from 2004 to 2011. (Source: Statista.com).

Year	2004	2006	2008	2010	2011
Total Number of Christmas Trees Sold in the U.S. (in millions)	27.10	28.60	28.20	27	30.80

a) Use your calculator to determine the equation of the *regression line, C*(*t*) where *t* represents the number of years since 2004.

Start by entering new *t* values for the table below based upon the number of years since 2004. The first few are done for you:

t = number of years since 2004	0	2			
Total number of Christmas trees sold in the U.S. (in millions)	27.10	28.60	28.20	27	30.80

Determine the regression equation in $y = ax + b$ form and write it here: ____________________
Round to three decimals as needed.

Rewrite the regression equation in $C(t) = at + b$ form and write it here: ____________________
Round to three decimals as needed.

b) Use the regression equation to determine C(3) and explain its meaning in the context of this problem.

c) Use the regression equation to predict the number of Christmas trees that will be sold in the year 2013. Write your answer as a complete sentence.

d) Identify the slope of the regression equation and explain its meaning in the context of this problem.

Section 4.5 – Multiple Ways to Determine the Equation of a Line

Problem 13	**WORKED EXAMPLE – Multiple Ways to Determine the Equation of a Line**

Determine if the data below represent a linear function. If so, use at least two different methods to determine the equation that best fits the given data.

x	1	5	9	13
y	75	275	475	675

Compute a few slopes to determine if the data are linear.

Between (1, 75) and (5, 275) $m = \frac{275-75}{5-1} = \frac{200}{4} = 50$

Between (5, 275) and (9, 475) $m = \frac{475-275}{9-5} = \frac{200}{4} = 50$

Between (9, 475 and 13, 675) $m = \frac{675-475}{13-9} = \frac{200}{4} = 50$

The data appear to be linear with a slope of 50.

Method 1 to determine Linear Equation – Slope Intercept Linear Form ($y = mx + b$):

Use the slope, m = 50, and one ordered pair, say (1, 75) to find the y-intercept
75 = 50(1) + b, so b=25.
Thus the equation is given by y = 50x + 25.

Method 2 to determine Linear Equation – Linear Regression:

Use the steps for Linear Regression to find the equation. The steps can be used even if the data are exactly linear.

Step 1: Go to STAT>EDIT>1:Edit
Step 2: Clear L1 by scrolling to L1 then press CLEAR then scroll back down one row
Step 3: Enter the values 1, 5, 9, 13 into the rows of L1 (pressing Enter between each one)
Step 4: Right arrow then up arrow to top of L2 and Clear L2 by pressing CLEAR then scroll back down
Step 5: Enter the values 75, 275, 475, 675 into the rows of L2 (pressing Enter between each one)
Step 6: Go to STAT>EDIT>CALC>4:LinReg (ax + b) then press ENTER repeatedly.
Step 7: Read the values a and b from the screen and use them to write the equation,
y = 50x + 25

Lesson 4 Practice Problems

Round to two decimal places unless stated otherwise.

Section 4.1: Review of Linear Functions

1. Edward the vampire can run at a speed of 70 miles per hour. His girlfriend Bella is 875 miles away from Edward visiting her mom in Phoenix. Edward decides to visit her. Edward's distance, *D*, from Bella t hours after he leaves for his trip can be modeled by the linear function $D(t) = -70t + 875$.

   a) Find the Vertical Intercept (y-int) of the function and write a sentence to interpret its meaning in the context of the problem.

   b) Find the Horizontal Intercept (x-int) of the function and write a sentence to interpret its meaning in the context of the problem.

   c) Evaluate $D(4)$ and write a sentence to interpret its meaning in the context of the problem.

   d) Find the *t* value for which $D(t) = 504$ and write a sentence to interpret its meaning in the context of the problem.

e) Is the function D increasing or decreasing? How do you know?

f) Determine the slope or rate of change of the function D (include the units). What does the rate of change represent in the context of the problem?

g) Determine the practical domain and practical range of this function. Assume that $t \geq 0$ and that Edward stops traveling when he reaches Bella in Phoenix.

Practical Domain: __________ $\leq t \leq$ ____________

Practical Range: __________ $\leq D(t) \leq$ ____________

Section 4.2: Average Rate of Change

2. For each of the following functions, determine if the function is linear. If it is linear, give the slope.

a)

x	−3	−2	−1	0	1	2	3
$f(x)$	2	4	8	16	32	64	128

b)

x	−3	−2	−1	0	1	2	3
$g(x)$	−2	0	2	4	6	8	10

c)

t	−4	−1	2	5	8	11	14
$s(t)$	28	19	10	1	−8	−17	−26

d)

x	−4	−2	0	2	4	6	8
$h(x)$	5	6	7	8	9	10	11

e)

n	−4	−1	5	6	8	9	12
$p(n)$	−4	−2	0	2	4	6	8

3. The data below represent the number of times your friend's embarrassing YouTube Video has been viewed per hour since you uploaded it. The data are exactly linear.

Time, t, in hours	0	1	2	3	4
Views, V, in thousands	0	6200	12400	18600	24800

a) Identify the Vertical Intercept (y-int) and average rate of change for the data.

b) Use your results from part a) to write the linear function that represents the data table. Use the indicated variables and proper function notation.

c) Use your function to determine the number of views after 8 hours. Write your final result as a complete sentence.

d) Use your function to determine how many hours until the number of views reaches 100,000. Round to the nearest whole hour. Write your final result as a complete sentence.

4. You adopted an adult cat four years ago. The data below represent your cat's weight for the four years she's lived with you. The data are exactly linear.

Time, t, in years	0	1	2	3	4
Weight, W, in pounds	6	7.25	8.5	9.75	11

a) Identify the Vertical Intercept (y-int) and average rate of change for the data.

b) Use your results from part a) to write the linear function that represents the data table. Use the indicated variables and proper function notation.

c) Use your function to determine how much the cat will weigh in year 8. Write your final result as a complete sentence.

d) Use your function to determine how many years it would take for your cat to reach 20 pounds. Round to the nearest whole year.

5. Data below represent how many pushups Tim can do in a minute at the start of a 5-week exercise program and each week thereafter.

Time, t, in weeks	0	1	2	3	4	5
Number of Pushups, P, in a minute	2	6	10	14	18	20

a) Compute the average rate of change for weeks 0 through 3. Be sure to include the unit of your answer.

b) Compute the average rate of change for weeks 1 through 4. Be sure to include the unit of your answer.

c) Compute the average rate of change for the whole 5 – week period (weeks 0 through 5). Be sure to include the unit of your answer.

d) What is the meaning of the average rate of change in this situation?

e) Do the data points in the table define a perfectly linear function? Why or why not?

6. You decided to save up for a vacation to Europe by throwing all your loose change in a large coffee can. After a few months, you discover that the jar is 2 inches full and contains $124.

   a) Determine the average rate of change, in $/inch (Dollars per inch), for the coffee can from when it was empty (0 inches) to when it was 2 inches deep.

   b) A month later, you check the can and find the change is 3 inches deep and adds up to $186. Find the average rate of change, in $/inch, for the coffee can from 0 inches to 3 inches.

   c) What is the *meaning* of the average rate of change in this situation?

   You do some additional calculations and create a table for the can of change.

d, depth of the change in inches	*V*, value of the can in dollars
0	0
2	124
3	186
5	310
10	620

   d) Use the information found so far to write an equation that describes this situation. Use function notation and the variable names from the table.

   e) You need $1000 for your vacation. In a complete sentence, state how deep the change has to be to reach your goal. Also, write the results as an ordered pair and in function notation.

Section 4.3: Scatterplots on the Graphing Calculator

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7. Use your graphing calculator to create of scatterplot of the data set shown below. Be sure to use an appropriate viewing window.

x	1	3	4	6	7	9	10
y	437	886	1097	1558	1768	2217	2437

In the space below, sketch what you see on your calculator screen, and write down the viewing window you used.

Viewing Window:

Xmin:__________
Xmax:__________
Ymin:__________
Ymax:__________

8. Use your graphing calculator to create of scatterplot of the data set shown below. Be sure to use an appropriate viewing window.

x	2	9	14	23	33	42
y	60.2	130.1	243.7	328.9	580.5	643.8

In the space below, sketch what you see on your calculator screen, and write down the viewing window you used.

Viewing Window:

Xmin:__________
Xmax:__________
Ymin:__________
Ymax:__________

Section 4.4: Linear Regression

9. The following table shows the number of newspaper subscriptions in Middletown, USA where *t* represents the number of years since 2002 (t = 0 in 2002) and *S(t)* represents the total subscriptions each year measured in thousands.

t (year)	0	2	4	6	8
(total subscriptions in 1000's)	448	372	198	145	45

a) Use your calculator to determine the equation of the *regression line*.

Determine the regression equation in $y = ax + b$ form and write it h

Rewrite the regression equation in $S(t) = at + b$ form and write it here:

b) Use your graphing calculator to create of scatterplot of the data set and the linear regression equation. Be sure to use an appropriate viewing window.

In the space below, sketch what you see on your calculator screen, and write down the viewing window you used.

Viewing Window:

Xmin:__________
Xmax:__________
Ymin:__________
Ymax:__________

c) Based on your graph above, do the data appear to be exactly linear, approximately linear or not linear? Explain.

d) What is the slope of your regression model for *S(t)* and what is its meaning in the context of this problem?

e) What is the Vertical Intercept (y-int) of your linear regression model for *S(t)* and what is its meaning in the context of the problem.

f) Use your linear regression equation to estimate the total number of subscriptions in 2007 (i.e. when *t* =5).. Show your computations here and your final result. Round to the nearest whole day.

g) Use your linear regression equation to estimate the total number of subscriptions in 2004. How does this value compare to the data value in the table?

h) Use your linear regression equation to estimate the year in which the circulation will be 100,000. Round to the closest whole year. (Reminder: *S(t)* is measured in thousands so solve *S(t) = 100*).

10. Scott is hiking the Appalachian Trail from Georgia to Maine. The distance of his hike is 2200 miles. It took Scott 123 days to complete the hike. The data below represent the distance, *D*, he had hiked *t* days after the start of his trip.

t (days hiking)	0	32	47	73	99	123
(distance in miles)	0	590	912	1212	1876	2200

a) Use your calculator to determine the equation of the *regression line*. (Round to 2 decimal places)

Determine the regression equation in $y = ax + b$ form and write it here:

Rewrite the regression equation in $D(t) = at + b$ form and write it here:

b) Use your graphing calculator to create a scatterplot of the data set and the linear regression equation. Be sure to use an appropriate viewing window.

In the space below, sketch what you see on your calculator screen, and write down the viewing window you used.

Viewing Window:

Xmin:__________
Xmax:__________
Ymin:__________
Ymax:__________

c) Based on your graph above, do the data appear to be exactly linear, approximately linear or not linear? Explain.

d) What is the slope of your regression model for *D(t)* and what is its meaning in the context of this problem?

e) Use your linear regression equation to estimate the total number of miles Scott has hiked in 50 days. Show your computations here and your final result.

f) Use your linear regression equation to estimate when Scott has hiked 1000 miles. Round to the nearest whole day.

11. Your turn. Create a story problem where the data change linearly and then create a table that has data points for that story.

a) Write the story problem.

b) Create a table for the story problem. Make sure you use Function Notation.

c) Compute the average rate of change for your data. Be sure to include units.

d) What is the meaning of the average rate of change in this situation?

e) Determine the Vertical Intercept (y-int) for your data. What is the meaning of this Vertical Intercept (y-int)?

f) Use the Vertical Intercept (y-int) and the rate of change to write the linear function model for the data. Use proper variable names and proper function notation.

g) Write a **read the data** question given the input. Write your question as a complete sentence and in function notation.

h) Write a **read the data** question given the output. Write your question as a complete sentence and in function notation.

i) Write a **read between the data (Interpolating the Data)** question given the input. Write your question as a complete sentence and in function notation.

j) Write a **read between the data (Interpolating the Data)** question given the output. Write your question as a complete sentence and in function notation.

k) Write a **read beyond the data (Extrapolating the Data)** question given the input. Write your question as a complete sentence and in function notation.

l) Write a **read beyond the data (Extrapolating the Data)** question given the output. Write your question as a complete sentence and in function notation.

Section 4.5: Multiple Ways to Determine the Equation of a Line

12. Sara is selling Girl Scout cookies. They cost $4 per box. The table below shows how much money Sara has collected, C, based on the number of days, t, she has been selling cookies.

t	0	2	3	7	12
y	0	96	144	336	576

a) Find the average rate of change for the following pairs of t values.

i. t = 0 and t = 2

ii. t = 2 and t = 7

iii. t = 3 and t = 12

b) Based on your answers to part a), is it possible that the data are exactly linear? Explain.

c) Create a scatterplot for the data on your calculator. In the space below, sketch what you see on your calculator screen, and write down the viewing window you used.

Viewing Window:

Xmin:__________
Xmax:__________
Ymin:__________
Ymax:__________

d) Based on your answer to part c), do the data appear to be exactly linear, approximately linear or not linear? Explain.

e) Use your graphing calculator to find a linear regression model for the data. Record the equation below. Also draw a sketch of the line with the scatterplot below.

Regression Equation:____________________________ (*form:* $C(t) = mt + b$)

Viewing Window:

Xmin:__________
Xmax:__________
Ymin:__________
Ymax:__________

f) Does the regression equation fall *exactly* on the data points, *approximately near* the data points or *not aligned* to the data points? Explain.

g) Explain the meaning of the slope of your regression equation. How does it compare to the average rate of change you found in *part a*?

13. Jose is recording the average daily temperature for his science class during the month of June in Phoenix, Arizona. The table below represents the average daily temperature *t* days after June 1st.

t	0	1	3	5	8	10
y	95	97	98	105	96	90

a) Find the average rate of change for the following pairs of t values.

i. $t = 0$ and $t = 1$

ii. $t = 1$ and $t = 5$

iii. $t = 3$ and $t = 10$

b) Based on your answers to part a, is it possible that the data are exactly linear? Explain.

c) Create a scatterplot for the data on your calculator. In the space below, sketch what you see on your calculator screen, and write down the viewing window you used.

Viewing Window:

Xmin:__________
Xmax:__________
Ymin:__________
Ymax:__________

d) Based on your answer to part c), do the data appear to be exactly linear, approximately linear or not linear? Explain.

e) Use your graphing calculator to find a linear regression model for the data. Record the equation below. Also draw a sketch of the line with the scatterplot below. Round to two decimal places.

Regression Equation:____________________________ (*form: $D(t) = mt + b$*)

Viewing Window:

Xmin:__________
Xmax:__________
Ymin:__________
Ymax:__________

f) Does the regression equation fall *exactly* on the data points, *approximately near* the data points or *not aligned* to the data points? Explain.

g) Do you think the regression model fits the data well? Explain.

Lesson 4 Assessment

1. Tamara is collecting donations for her local food bank. The data below represents the pounds of food, *P*, in the food bank t days after November 1st.

t	0	1	3	6	12	15
y	123	133	152	184	147	274

a) Find the average rate of change for the following pairs of t values.

i. t = 0 and t = 1

ii. t = 1 and t = 6

iii. t = 3 and t = 15

b) Based on your answers to part a, is it possible that the data are exactly linear? Explain.

c) Create a scatterplot for the data on your calculator. In the space below, sketch what you see on your calculator screen, and write down the viewing window you used.

Viewing Window:

Xmin:__________
Xmax:__________
Ymin:__________
Ymax:__________

d) Based on your answer to part c) do the data appear to be exactly linear, approximately linear or not linear? Explain.

e) Use your graphing calculator to find a linear regression model for the data. Round to two decimal places. Record the equation below. Also draw a sketch of the line with the scatterplot on the previous page.

Regression Equation:____________________________ (*form:* $P(t) = mt + b$)

f) Using the regression equation, P(t) in part e), identify the slope and explain its meaning in the context of the problem by writing a complete sentence.

2. The data below represents the cost to tow a car m miles. The data are exactly linear.

Miles, m	0	10	20	30	40
Cost, C, in dollars	50	300	550	800	1050

a) Identify the Vertical Intercept (y-int) and explain its meaning in the context of the problem by writing a complete sentence.

b) Find the average rate of change and explain its meaning in the context of the problem by writing a complete sentence.

c) Use your results from part a) to write the linear function that represents the data table. Use the indicated variables and proper function notation.

Lesson 5 – Introduction to Quadratic Functions

We are leaving linear functions behind and entering an entirely different world. As you work through this lesson, you will learn to identify quadratic functions and their graphs (called parabolas). You will learn the important parts of the parabola including the direction of opening, the vertex, intercepts, and axis of symmetry.

You will use graphs of quadratic functions to solve equations and, finally, you will learn how to recognize all the important characteristics of quadratic functions in the context of a specific application. Even if a problem does not ask you to graph the given quadratic function or equation, doing so is always a good idea so that you can get a visual feel for the problem at hand.

Lesson Topics

Mini-Lesson 5

Section 5.1 – Characteristics of Quadratic Functions

A QUADRATIC FUNCTION is a function of the form

$$f(x) = ax^2 + bx + c\ ,\ a \neq 0$$

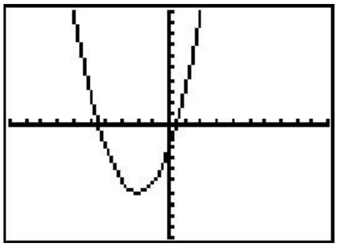
This form of a quadratic function is called the **GENERAL FORM**.

Characteristics Include:
- Degree 2 polynomial
- Three distinct terms each with its own coefficient:
    - An x^2 term with coefficient a
    - An x term with coefficient b
    - A constant term, c
    - Note: If any term is missing, the coefficient of that term is 0
- The graph of this function is called a "parabola", is shaped like a "U", and opens either up or down (concave up/concave down)
- a determines which direction the parabola opens ($a > 0$ opens up (concave up), $a < 0$ opens down (concave down))
- c is the Vertical Intercept (y-int) with coordinates (0, c)
- Domain: $(-\infty, \infty)$

Problem 1	**WORKED EXAMPLE – GRAPH QUADRATIC FUNCTIONS**

Given the Quadratic Function $f(x) = x^2 + 4x - 2$, complete the table and generate a graph of the function.

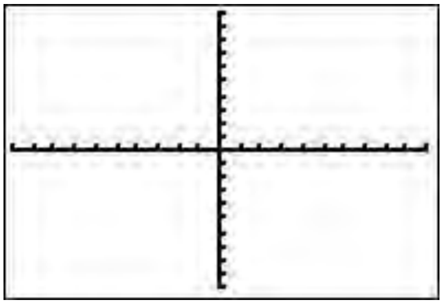
Identity the coefficients a, b, c	$a = 1$, $b = 4$, $c = -2$
Which direction does the parabola open?	$a = 1$ which is greater than 0 so parabola opens up
What is the Vertical Intercept (y-int)?	c = –2 so Vertical Intercept (y-int) = (0, –2)



Problem 2	**MEDIA EXAMPLE – GRAPH QUADRATIC FUNCTIONS**

Given the Quadratic Function $f(x) = x^2 - 2x + 3$, complete the table and generate a graph of the function.

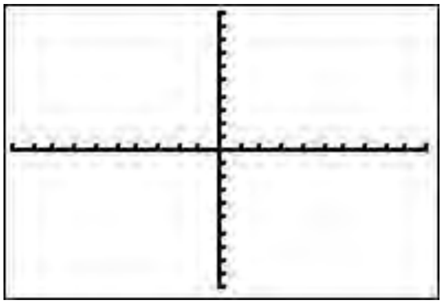
Identity the coefficients *a, b, c*	
Which direction does the parabola open? Why?	
What is the Vertical Intercept (y-int)?	



Problem 3	**YOU TRY – GRAPH QUADRATIC FUNCTIONS**

Given the Quadratic Function $f(x) = 2x^2 - 5$, complete the table and generate a graph of the function.

Identity the coefficients *a, b, c*	
Which direction does the parabola open? Why?	
What is the Vertical Intercept (y-int)? Plot and label on the graph.	



Given a quadratic function, $f(x) = ax^2 + bx + c$:

The VERTEX is the lowest or highest point (ordered pair) of the parabola

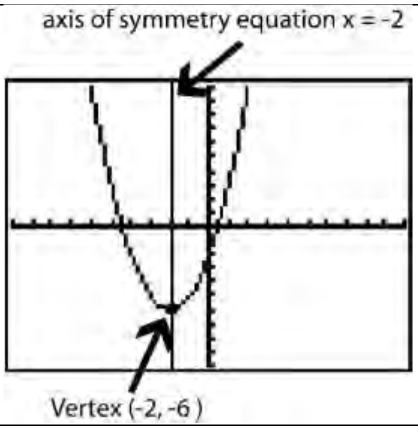
- To find the input value, identify coefficients *a* and *b* then compute $x = -\frac{b}{2a}$
- Plug this input value into the function to determine the corresponding output value, (i.e. evaluate $f\left(-\frac{b}{2a}\right)$)
- The Vertex is always written as an ordered pair. Vertex = $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

The AXIS OF SYMMETRY is the vertical line that intersects the Vertex, dividing the parabola in half. The axis of symmetry is represented as the equation:

- $x = -\frac{b}{2a}$

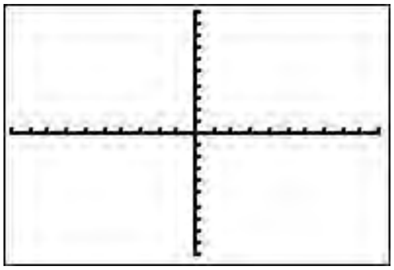
Problem 4 **WORKED EXAMPLE – Quadratic Functions: Vertex/Axis Of Symmetry**

Given the Quadratic Function $f(x) = x^2 + 4x - 2$, complete the table below.

Identity the coefficients *a, b, c*	$a = 1, \quad b = 4, \quad c = -2$
Identify the Axis of Symmetry Equation.	Axis of Symmetry: $x = -2$
Determine the coordinates of the Vertex.	Input Value Output Value $x = -\frac{b}{2a}$ $f(-2) = (-2)^2 + 4(-2) - 2$ $x = -\frac{(4)}{2(1)}$ $f(-2) = 4 - 8 - 2$ $f(-2) = -6$ $x = -2$ Vertex Ordered Pair: (–2, –6)
Sketch the Graph	

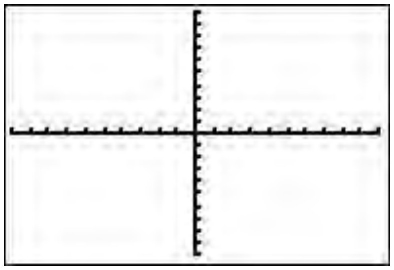
Problem 5	**MEDIA EXAMPLE – Quadratic Functions: Vertex/Axis Of Symmetry**

Given the Quadratic Function $f(x) = x^2 - 2x + 3$, complete the table, generate a graph of the function, and plot/label the vertex and axis of symmetry on the graph.

Identify the coefficients *a, b, c*	
Identify the Axis of Symmetry Equation.	
Determine the coordinates of the Vertex.	
Graph of the function. Plot/label the vertex and axis of symmetry on the graph.	

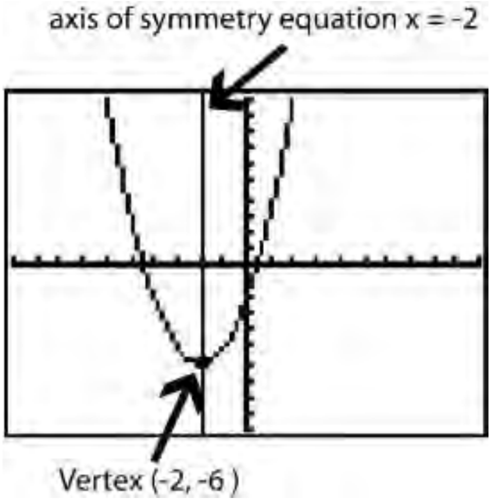
Problem 6	**YOU TRY – Quadratic Functions: Vertex/Axis Of Symmetry**

Given the Quadratic Function $f(x) = 2x^2 - 5$, complete the table, generate a graph of the function, and plot/label the vertex and axis of symmetry on the graph.

Identify the coefficients *a, b, c*	
Identify the Axis of Symmetry Equation.	
Determine the coordinates of the Vertex.	
Graph of the function. Plot/label the vertex and axis of symmetry on the graph.	

Problem 7	WORKED EXAMPLE – Quadratic Functions: Domain and Range

Determine the Domain and Range of the Quadratic Function $f(x) = x^2 + 4x - 2$



Domain of $f(x)$:

All real numbers. $-\infty < x < \infty$ $(-\infty, \infty)$

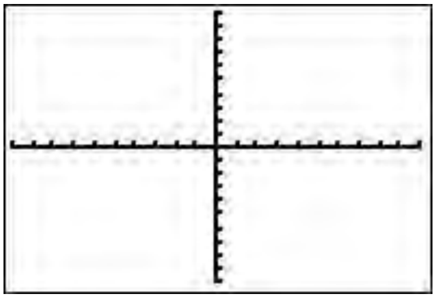
Range of $f(x)$:

Since the parabola opens upwards, the vertex (–2, –6) is the *lowest* point on the graph.

The Range is therefore $-6 \le f(x) < \infty$, or $[-6, \infty)$

Problem 8	MEDIA EXAMPLE – Quadratic Functions: Domain and Range

Determine the Domain and Range of $f(x) = -2x^2 - 6$.

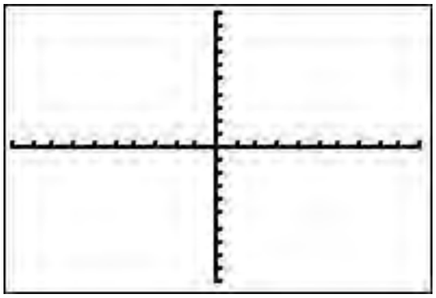


Domain of $f(x)$:

Range of $f(x)$:

Problem 9	YOU TRY – Quadratic Functions: Domain and Range

Determine the Domain and Range of $f(x) = 2x^2 - 5$. Sketch the graph and label the vertex.

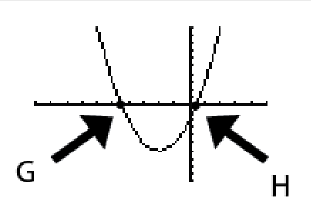


Vertex as an ordered pair:

Domain of $f(x)$:

Range of $f(x)$:

Finding Horizontal Intercept (x-int's) of a Quadratic Function



The quadratic function, $f(x) = ax^2+bx+c$, will have Horizontal Intercept (x-int)s when the graph crosses the x-axis (i.e. when $f(x) = 0$). These points are marked on the graph above as G and H. To find the coordinates of these points, what we are really doing is solving the equation $ax^2+bx+c = 0$. At this point, we will use the following general calculator process. In the next lesson, we will learn other methods for solving these equations.

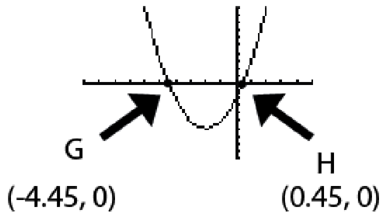
Calculator Process to solve $ax^2+bx+c = 0$

1. Press Y= then enter $f(x)$ into Y1
2. Enter 0 into Y2
3. Use the graphing/intersection method once to determine G and again to determine H.

Problem 10	**WORKED EXAMPLE – Finding Horizontal Intercept (x-int)s of a Quadratic Function**

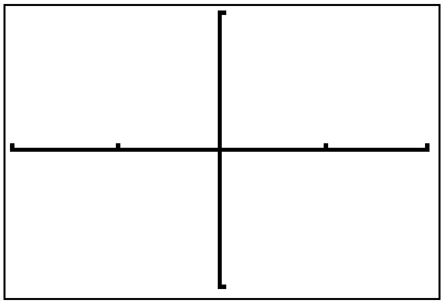
Find the Horizontal Intercept(s) (x-ints) of $f(x) = x^2 + 4x - 2$ and plot/label them on the graph.

1. Press Y= then enter $x^2 + 4x - 2$ into Y1
2. Enter 0 into Y2
3. Use the graphing/intersection method once to determine G as (–4.45, 0). You may have to move your cursor close to G during the "First Curve?" part.
4. Use the graphing/intersection method again to determine H as (0.45, 0). You may have to move your cursor close to H during the "First Curve?" part.



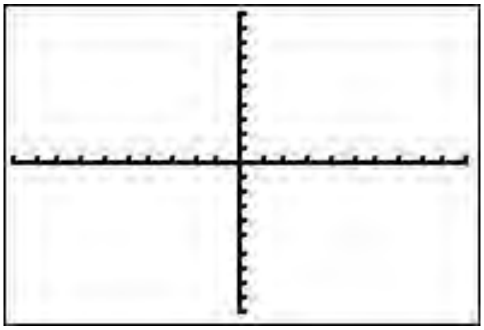
Problem 11	**MEDIA EXAMPLE – Finding Horizontal Intercept(s) (x-int) of a Quadratic Function**

Given the Quadratic Function $f(x) = x^2 - x - 6$, find the Horizontal Intercept(s) (x-int) and plot/label them on the graph.



Problem 12	**YOU TRY – Finding Horizontal Intercept (x-int) of a Quadratic Function**

Given the Quadratic Function $f(x) = 2x^2 - 5$, find the vertex, Vertical Intercept (y-int), and Horizontal Intercept(s) (x-ints). Plot and label all of these points on the graph. Round your values to two decimals.



Axis of Symmetry :____________________

Vertex: (______, ______)

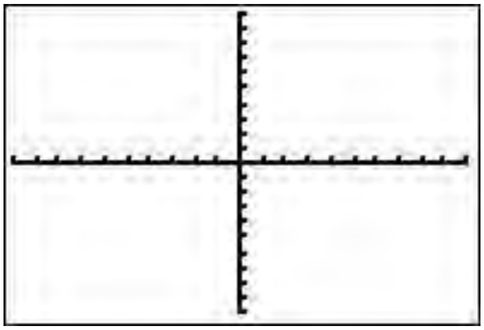
Vertical Intercept (y-int): (______, ______)

Horizontal Intercept (x-int's):

(______, ______) and (______, ______)

Problem 13	**YOU TRY – Finding Horizontal Intercept (x-int) of a Quadratic Function**

Given the Quadratic Function $f(x) = -x^2 + 2x + 5$, find the vertex, Vertical Intercept (y-int), and Horizontal Intercept(s) (x-ints). Plot and label all of these points on the graph. Round your values to two decimals.



Axis of Symmetry :____________________

Vertex: (______, ______)

Vertical Intercept (y-int): (______, ______)

Horizontal Intercept (x-int's):

(______, ______) and (______, ______)

A QUADRATIC **EQUATION** in GENERAL FORM is an equation of the form

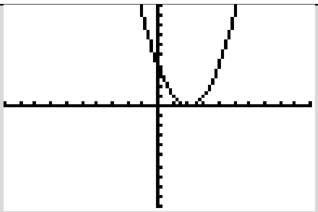
$$ax^2 + bx + c = 0 \quad a \neq 0$$

If the quadratic equation $ax^2 + bx + c = 0$ has real number solutions x_1 and x_2, then the x-intercepts of $f(x) = ax^2 + bx + c$ are $(x_1, 0)$ and $(x_2, 0)$.

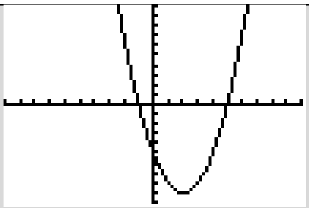
Note that if a parabola does not cross the x-axis, then its solutions lie in the complex number system and we say that it has *no real-valued x-intercepts*.

There are three possible cases for the number of solutions to a quadratic equation in general form.

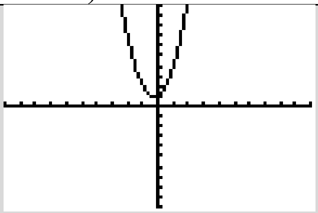
CASE 1: One, repeated, real number solution

The parabola touches the x-axis in *just one* location (i.e. only the vertex touches the x-axis)	

CASE 2: Two unique, real number solutions

The parabola crosses the x-axis at *two* unique locations.	

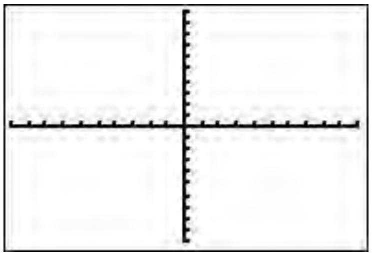
CASE 3: No real number solutions (but two Complex number solutions)

The parabola does NOT touch or cross the x-axis.	

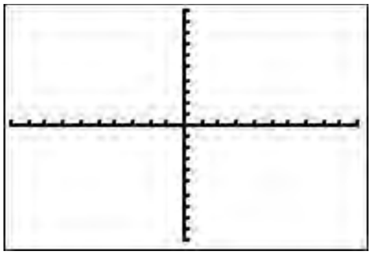
Problem 14	**MEDIA EXAMPLE – HOW MANY AND WHAT KIND OF SOLUTIONS?**

Use your graphing calculator to help you determine the number and type of solutions to each of the quadratic equations below. Begin by putting the equations into general form. Draw an accurate sketch of the parabola in an appropriate viewing window. IF your solutions are real number solutions, use the graphing INTERSECT method to find them. Use proper notation to write the solutions and the Horizontal Intercepts (x-int(s)) of the parabola. Label the intercepts on your graph.

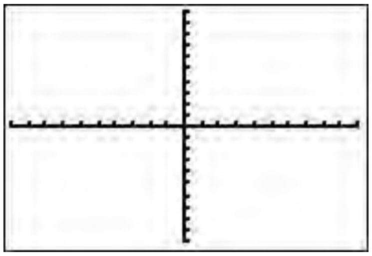
a) $x^2 - 10x + 25 = 0$



b) $-2x^2 + 8x - 3 = 0$



c) $3x^2 - 2x = -5$



Problem 15 YOU TRY – HOW MANY AND WHAT KIND OF SOLUTIONS?

Use your graphing calculator to help you determine the number and type of solutions to each of the quadratic equations below. Begin by putting the equations into general form. Draw an accurate sketch of the parabola in an appropriate viewing window (the vertex, Vertical Intercept (y-int), and any Horizontal Intercepts (x-int(s)) should appear on the screen). IF your solutions are real number solutions, use the graphing INTERSECT method to find them. Use proper notation to write the solutions and the Horizontal Intercepts (x-int) of the parabola. Label the intercepts on your graph.

a) $-x^2 - 6x - 9 = 0$

Xmin = ________ Ymin = ________

Xmax = ________ Ymax = ________

Number of Real Solutions:__________

Real Solutions: ____________________

Horizontal Int(s):___________________

b) $3x^2 + 5x + 20 = 0$

Xmin = ________ Ymin = ________

Xmax = ________ Ymax = ________

Number of Real Solutions:__________

Real Solutions: ____________________

Horizontal Int(s):___________________

c) $2x^2 + 5x = 7$

Xmin = ________ Ymin = ________

Xmax = ________ Ymax = ________

Number of Real Solutions:__________

Real Solutions: ____________________

Horizontal Int(s):___________________

Section 5.2 – Solving Quadratic Equations Graphically

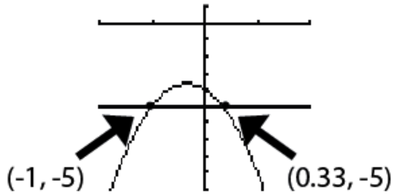
A quadratic equation of the form $ax^2+bx+c = d$ can be solved in the following way using your graphing calculator:

1. Go to Y=
2. Let Y1= $ax^2 + bx + c$
3. Let Y2 = d
4. Graph the two equations. You may need to adjust your window to be sure the intersection(s) is/are visible.
5. For each intersection, use 2nd>Calc>Intersect. Follow on-screen directions to designate each graph then determine intersection (hitting Enter each time).
6. Solution(s) to the equation are the intersecting x-values

NOTE: The Intersection method will provide us only with approximate solutions to a quadratic equation when decimal solutions are obtained. To find EXACT solution values, you will need to use the Quadratic Formula. This will be covered in the next lesson.

Problem 16 **WORKED EXAMPLE – Solve Quadratic Equations Graphically**

Solve the equation $–3x^2 – 2x – 4 = –5$ by graphing.



There are two intersection points. Follow the process above to find the intersections (–1, –5) and (0.33, –5). Solutions to the equation are $x = –1$ and $x = 0.33$.

Problem 17 **MEDIA EXAMPLE – Solve Quadratic Equations Graphically**

Solve $x^2 – 10x +1 = 4$. Plot and label the graphs and intersection points that are part of your solution process. Identify the final solutions clearly. Round to 2 decimals.

Xmin:_______

Xmax: _______

Ymin: _______

Ymax: _______

Problem 18	**YOU TRY – Solve Quadratic Equations Graphically**

a) Solve $2x^2 - 5 = 6$. Plot and label the graphs and intersection points that are part of your solution process. Round your answer to the nearest hundredth. Identify the final solutions clearly.

Xmin:_______

Xmax: _______

Ymin: _______

Ymax: _______

Solution(s):____________________________

b) Solve $x^2 + 9x - 18 = 32$. Plot and label the graphs and intersection points that are part of your solution process. Round your answer to the nearest hundredth. Identify the final solutions clearly.

Xmin:_______

Xmax: _______

Ymin: _______

Ymax: _______

Solution(s):____________________________

Section 5.3 – Applications of Quadratic Functions

A large number of quadratic applications involve launching objects into the sky (arrows, baseballs, rockets, etc…) or throwing things off buildings or spanning a distance with an arched shape. While the specifics of each problem are certainly different, the information below will guide you as you decipher the different parts.

HOW TO SOLVE QUADRATIC APPLICATION PROBLEMS

1. Draw an accurate graph of the function using first quadrant values only. Label the x-axis with the input quantity and units. Label the y-axis with the output quantity and units.
2. Identify, plot, and label the Vertical Intercept (y-int).
3. Identify, plot, and label the vertex.
4. Identify, plot, and label the positive Horizontal Intercept (x-int)(s) (usually, there is only one Horizontal Intercept (x-int) that we care about…if both are needed for some reason, then plot them both and include negative input values in your graph for part 1).
5. Once you have done steps 1 – 4, THEN read the specific questions you are asked to solve.

Questions that involve the Vertical Intercept (y-int) $(0, c)$:

- How high was the object at time t = 0? c
- What was the starting height of the object? c

Questions that involve the vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$:

- How high was the object at its highest point? $f\left(-\frac{b}{2a}\right)$
- What was the max height of the object? $f\left(-\frac{b}{2a}\right)$
- How long did it take the object to get to its max height? $x = -\frac{b}{2a}$
- What is the practical range of this function? $0 \leq f(x) \leq f\left(-\frac{b}{2a}\right)$

Questions that involve (usually) the positive Horizontal Intercept (x-int) $(x_2, 0)$:

- When did the object hit the ground? x_2
- What is the practical domain of this function? $0 \leq x \leq x_2$
- How long did it take the object to hit the ground? x_2
- How far was the object from the center? x_2

Problem 19	**WORKED EXAMPLE –APPLICATIONS OF QUADRATIC FUNCTIONS**

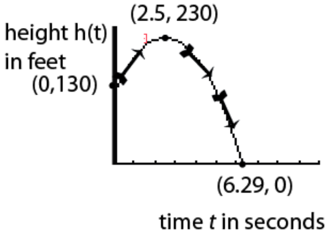
The function $h(t) = -16t^2 + 80t + 130$, where $h(t)$ is height in feet, models the height of an arrow shot into the sky as a function of time (seconds).

Before even LOOKING at the specific questions asked, find the following items and plot/label the graph.

1. Identify the Vertical Intercept (y-int). *(0, 130) since c = 130.*
2. Determine the vertex.

   The input value of the vertex is $x = -\frac{b}{2a} = -\frac{80}{2(-16)} = 2.5$.

   The corresponding output value is $f(-\frac{b}{2a}) = f(2.5) = -16(2.5)^2 + 80(2.5) + 130 = 230$
3. Determine the positive Horizontal Intercept (x-int) – *using the process discussed in earlier examples, we want to solve* $-16t^2 + 80t + 130=0$*. Using the intersect method, the positive Horizontal Intercept (x-int) is (6.29, 0).*
4. Draw an accurate graph of the function using first quadrant values only. Label the horizontal axis with the input quantity and units. Label the vertical axis with the output quantity and units. *Label the vertex and intercepts.*



QUESTIONS TO ANSWER NOW:

a) After how many seconds does the arrow reach its highest point?

The input value of the vertex is 2.5. So, the arrow reaches its highest point after 2.5 seconds.

b) How high is the arrow at its highest point?

The output value of the vertex is 230. So, the arrow is 230 feet above the ground at its highest point.

c) After how many seconds does the arrow hit the ground?

The Horizontal Intercept (x-int) is (6.29,0). The arrow will hit the ground after 6.29 seconds.

d) What is the practical domain of this function?

Time starts at 0 seconds and goes until the arrow hits the ground. So, practical domain is $0 \le t \le 6.29$ *seconds.*

e) What is the practical range of this function?

The arrow passes through all height values from 0 (when it hits the ground) to its max height of 230 ft. So, practical range is $0 \le h(t) \le 230$ *feet.*

f) What does the Vertical Intercept (y-int) represent?

The Vertical Intercept (y-int) represents the height of the arrow at time t = 0. Thus, the arrow starts at 130 feet off the ground.

Problem 20	**MEDIA EXAMPLE – APPLICATIONS OF QUADRATIC FUNCTIONS**

A train tunnel is modeled by the quadratic function $h(x) = –0.35x^2 + 25$, where x is the distance, in feet, from the center of the tracks and $h(x)$ is the height of the tunnel, also in feet. Assume that the high point of the tunnel is directly in line with the center of the train tracks.

a) Draw a complete diagram of this situation. Find and label each of the following: vertex, Horizontal Intercept (x-int) (positive side), and Vertical Intercept (y-int).

b) How wide is the base of the tunnel?

c) A train with a flatbed car 6 feet off the ground is carrying a large object that is 15 feet high. How much room will there be between the top of the object and the top of the tunnel?

Problem 21	**YOU TRY – APPLICATIONS OF QUADRATIC FUNCTIONS**

A toy rocket is shot straight up into the air. The function $H(t) = -16t^2 + 128t + 3$ gives the height (in feet) of a rocket after *t* seconds. Round answers to two decimal places as needed. All answers must include appropriate units of measure.

a) Draw a complete diagram of this situation. Find and **label** each of the following: vertex, Horizontal Intercept (x-int) (positive side), and Vertical Intercept (y-int).

b) How long does it take for the rocket to reach its maximum height? Write your answer in a complete sentence.

c) What is the maximum height of the rocket? Write your answer in a complete sentence.

d) How long does it take for the rocket to hit the ground? Write your answer in a complete sentence.

e) Identify the Vertical Intercept (y-int). Write it as an ordered pair and interpret its meaning in a complete sentence.

f) Determine the practical domain of *H*(*t*). Use inequality notation and include units.

g) Determine the practical range of *H*(*t*). Use inequality notation and include units.

Section 5.4 – Standard (Vertex) Form of Quadratic Functions – Graphical Approach

Given the quadratic function in general form $f(x) = ax^2 + bx + c$ where $a \neq 0$ and a, b, c are real numbers we can place the function into standard form using the algebraic method of completing the square to obtain:

Standard (Vertex) Form of Quadratic Functions

$$f(x) = a(x-h)^2 + k \ , \ a \neq 0$$

where

$Vertex = (h,k)$

Equation of the Axis of Symmetry is $x = h$.

Domain

Any quadratic function in the form $f(x) = a(x-h)^2 + k$ has the domain $(-\infty, \infty)$.

Range

Any quadratic function in the form $f(x) = a(x-h)^2 + k$ the range falls into one of two cases:

Case I: If $a > 0$ then the range of the quadratic function is $[k, \infty)$ and the function has a minimum value of $y = k$.

Case II: If $a < 0$ then the range of the quadratic function is $(-\infty, k]$ and the function has a maximum value of $y = k$.

Problem 22	**WORKED EXAMPLE – Standard Form of Quadratic Functions**

A) Use your graphing calculator to find the vertex, equation of the axis of symmetry, horizontal intercept(s), vertical intercept, domain, and range of the function $f(x) = -2(x-3)^2$. Round to two decimal places as needed.

Solution:

$Vertex = (h,k) = (3,0)$

Equation of the Axis of Symmetry: $x = 3$

$Domain = (-\infty, \infty)$

Since $a = -2, a < 0$ Circle One: Opens Up or [✓ Opens Down]

Circle One: [✓ Max] or Minimum

$Range = (-\infty, 0]$

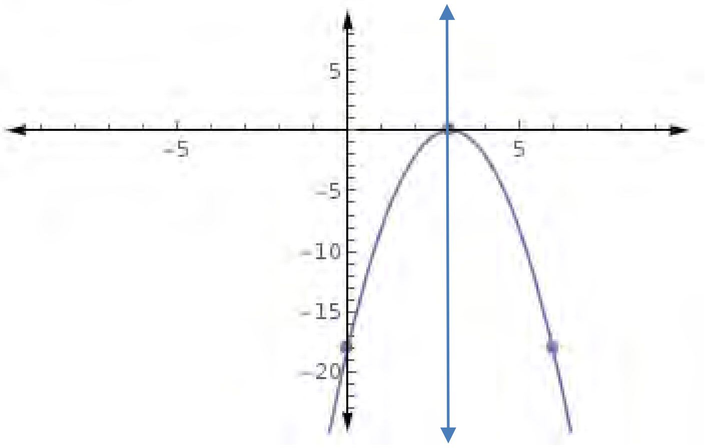
Horizontal Intercept (using intersect method on calc) : $(3, 0)$

Vertical Intercept: $f(0) = -2(0-3)^2 = -2(-3)^2 = -18$

(0, -18)

.

B) Draw a graph of the function labeling the Vertex, Axis of Symmetry, and two other points.



Problem 23	**YOU TRY – Standard Form of Quadratic Functions**

A) Use your graphing calculator to find the vertex, equation of the axis of symmetry, horizontal intercept(s), vertical intercept, domain, and range of the function $f(x)=\frac{1}{2}(x-2)^2-3$. Round to two decimal places as needed.

Vertex:____________

Equation of the Axis of Symmetry:_____________

Domain:____________

Since a=_____ Circle One: Opens Up or Opens Down

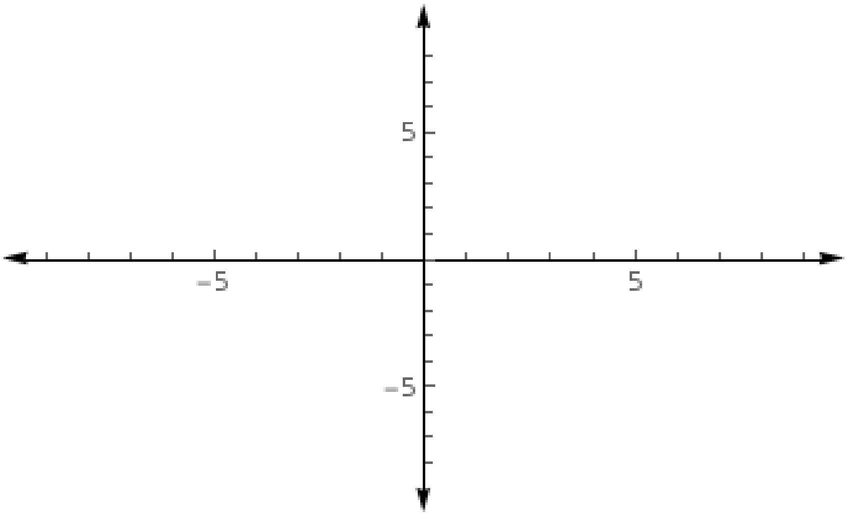
Circle One: Maximum or Minimum

Range:_____________

Horizontal Intercept(s) (x-int) (Intersect Method):

Vertical Intercept (y-int):

B) Draw a graph of the function labeling the Vertex, Axis of Symmetry, and two other points.



Problem 24	**YOU TRY – Standard Form of Quadratic Functions**

A) Use your graphing calculator to find the vertex, equation of the axis of symmetry, horizontal intercept(s), vertical intercept, domain, and range of the function $f(x) = -(x+2)^2 - 3$. Round to two decimal places as needed.

Vertex:____________

Equation of the Axis of Symmetry:_____________

Domain:____________

Since a=_____ Circle One: Opens Up or Opens Down

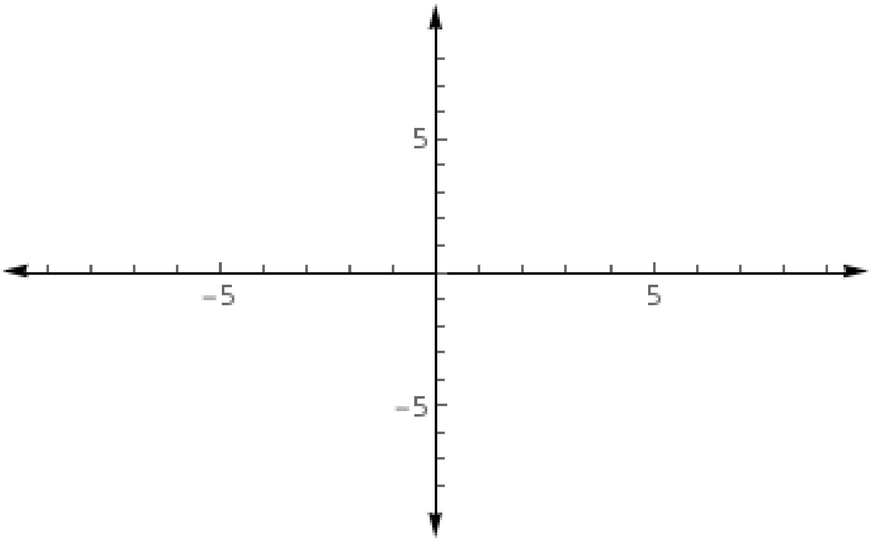
Circle One: Maximum or Minimum

Range:_____________

Horizontal intercept(x-int(s)) (Intersect Method):_________

Vertical intercept (y-int):__________

B) Draw a graph of the function labeling the Vertex, Axis of Symmetry, and two other points.



Section 5.5 – Quadratic Regression

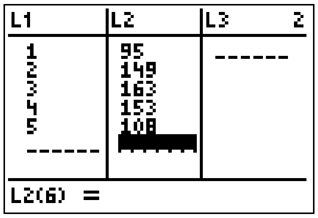
Problem 25	**WORKED EXAMPLE–Quadratic Regression**

The table below shows the height, H in feet, of an arrow t seconds after being shot.

t	1	2	3	4	5
$H(t)$	95	149	163	153	108

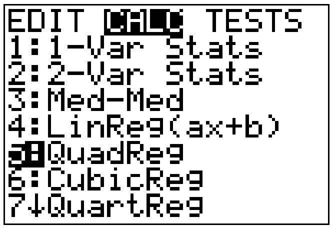
Use the Quadratic Regression feature of your calculator to generate a mathematical model for this situation. Round to three decimals.

- Press STAT>EDIT>ENTER to show data entry area. The STAT button is on the second row, third column.



```
L1     L2     L3     2
1      95     ------
2      149
3      163
4      153
5      108
------
L2(6) =
```

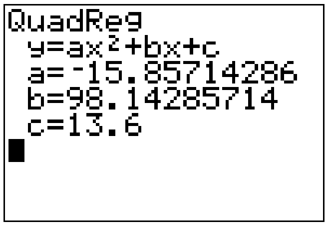
- Press STAT > CALC > 5:QuadReg



```
EDIT CALC TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7↓QuartReg
```

- Thus, your quadratic function (with values rounded as the problem indicates) is $y = -15.857x^2 + 98.143x + 13.6$

  Convert this to function notation with the appropriate variables to get $H(t) = -15.857t^2 + 98.143t + 13.6$



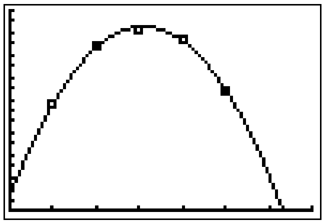
```
QuadReg
 y=ax²+bx+c
 a=-15.85714286
 b=98.14285714
 c=13.6
```

- Enter your function into Y1 to obtain a graph of your data and regression line.

Use viewing window

xmin=0 xmax=7

ymin=0 ymax=180



Problem 26	**YOU TRY – Quadratic Regression**

The table below shows the height, H in feet, of a golf ball t seconds after being hit.

t	1	2	3	4	5
$H(t)$	81	131	148	130	87

a) Use the Quadratic Regression feature of your calculator to generate a mathematical model for this situation. Use function notation with the appropriate variables.

b) Use your model to predict the height of the golf ball at 5 seconds. Round your answer to the nearest hundredth. How does this compare to the value in the data table?

c) Using your model to determine the maximum height of the golf ball. Round your answer to the nearest hundredth.

d) Use your model and the calculator (Intersect Method) to determine how long it will take the golf ball to hit the ground. Round your answer to the nearest hundredth.

e) Use your model to determine the practical domain and practical range for this situation (using inequality notation).

Practical Domain	Practical Range
______________________________	______________________________

f) Use your graphing calculator to create a graph of the data *and* the function H(t) on the same screen. Use the practical domain and range to determine an appropriate viewing window. In the space below, sketch what you see on your calculator screen, and indicate the window you used.

Xmin:_________

Xmax:_________

Ymin:_________

Ymax:_________

Lesson 5 Practice Problems

Round to two decimal places unless stated otherwise.

Section 5.1: Characteristics of Quadratic Functions

1. For each of the following quadratic functions:
   - Identify the coefficients *a, b, c*
   - Identify the vertical-intercept.
   - Mark and label the Vertical Intercept (y-int) on the graph.
   - Determine if the parabola opens up or down and state why.
   - Graph the function on your calculator. Draw the graph neatly below.

a) $f(x) = 2x^2 - 4x - 4$

$a =$ _________ $b =$ _________ $c =$ _________

Vertical Intercept (y-int): __________

Which direction does this parabola open? Why?

b) $f(x) = -x^2 + 6x - 4$

$a =$ _________ $b =$ _________ $c =$ _________

Vertical Intercept (y-int): __________

Which direction does this parabola open? Why?

c) $f(x) = 2x^2 - 6x + 4$

$a =$ _________ $b =$ _________ $c =$ _________

Vertical Intercept (y-int): __________

Which direction does this parabola open? Why?

d) $f(x) = x^2 - 3x$

$a =$ _________ $b =$ _________ $c =$ _________

Vertical Intercept (y-int): __________

Which direction does this parabola open? Why?

e) $f(x) = \dfrac{x^2}{2} - 3$

$a =$ _________ $b =$ _________ $c =$ _________

Vertical Intercept (y-int): __________

Which direction does this parabola open? Why?

2. For each of the following quadratic functions (Show your work):
   - Calculate the vertex *by hand* and write it as an ordered pair.
   - Determine the axis of symmetry and write it as a linear equation (x = #).

	Function	$x = -\dfrac{b}{2a}$	$f\left(-\dfrac{b}{2a}\right)$	Vertex	Axis of Symmetry
a)	$f(x) = -2x^2 + 2x - 3$				
b)	$g(x) = \dfrac{x^2}{2} - 3x + 2$				
c)	$f(x) = -x^2 + 3$				
d)	$p(t) = 4t^2 + 2t$				
e)	$h(x) = 3x^2$				

3. Use your graphing calculator to help you determine the number and type of solutions to each of the quadratic equations below. Round to two decimal places.
   - Draw an accurate sketch of the graphs indicating the window you used. The vertex and any intercepts or intersection points must appear on the screen.
   - IF your solutions are real number solutions, use the graphing INTERSECT method to find them.
   - Use proper notation to write the solutions and label the horizontal intercept(s) on the graph.
   - Label the intercepts on your graph.

a) $x^2 - 6x + 9 = 0$

Xmin = ________ Ymin = ________

Xmax = ________ Ymax = ________

Number of Real Solutions:__________

Real Solutions: ___________________

b) $5x^2 + 4x - 5 = 0$

Xmin = ________ Ymin = ________

Xmax = ________ Ymax = ________

Number of Real Solutions:__________

Real Solutions: ___________________

c) $2x^2 - 4x = 3$

Xmin = ________ Ymin = ________

Xmax = ________ Ymax = ________

Number of Real Solutions:__________

Real Solutions: ___________________

d) $3x^2 + 6x + 4 = 0$

Xmin = ________ Ymin = ________

Xmax = ________ Ymax =________

Number of Real Solutions:__________

Real Solutions: ___________________

e) $3x^2+5=6x$

Xmin = ________ Ymin = ________

Xmax = ________ Ymax = ________

Number of Real Solutions:__________

Real Solutions: ___________________

f) $-7x^2=12x-4$

Xmin = ________ Ymin = ________

Xmax = ________ Ymax =________

Number of Real Solutions:__________

Real Solutions: ___________________

4. Use problem 2 to help you complete the table.

	Function	Domain	Range	Horizontal Intercept (x-int)(s) (if any)
a)	$f(x)=-2x^2+2x-3$			
b)	$g(x)=\frac{x^2}{2}-3x+2$			
c)	$f(x)=-x^2+3$			
d)	$p(t)=4t^2+2t$			
e)	$h(x)=3x^2$			

5. For each quadratic function:
   - Graph the function on your calculator using an appropriate viewing window. Draw the graph neatly below.
   - Label the Vertical Intercept (y-int).
   - Determine the vertex. Mark and label the vertex on the graph.
   - Determine the Horizontal Intercept (x-int(s)) (if they exist) and label them on the graph (Round to two decimal places)

a) $f(x) = -2x^2 + 6x + 3$

Vertical Intercept (y-int): __________

Horizontal Intercept (x-int)(s):

____

Vertex: ___________

b) $f(x) = \frac{x^2}{2} - 2x$

Vertical Intercept (y-int): __________

Horizontal Intercept (x-int)(s):

____

Vertex: ___________

c) $f(x) = 5x^2 + 4$

Vertical Intercept (y-int): __________

Horizontal Intercept (x-int)(s):

____

Vertex: ___________

Section 5.2: Solving Quadratic Equations Graphically

6. Solve each equation using your calculator. Draw the graph and plot/label the point(s) of intersection. Clearly identify the final solution(s).

a) $x^2 - x - 6 = 0$

Solution(s):______________________________

b) $x^2 - 9x + 10 = -4$

Solution(s):______________________________

c) $x^2 - 8 = 1$

Solution(s):______________________________

7. Solve each equation using your calculator. Draw the graph and plot/label the point(s) of intersection. Clearly identify the final solution(s).

a) $-x^2+6x-4=-10$

Solution(s):______________________________

b) $\frac{3}{2}x^2-6x+6=10$

Solution(s):______________________________

c) $5x^2+\frac{x}{2}-5=8$

Solution(s):______________________________

Section 5.3: Applications of Quadratic Functions

8. The function $h(t) = -0.2t^2 + 1.3t + 15$, where *h(t)* is height in feet, models the height of an "angry bird" shot into the sky as a function of time (seconds).

   a) Draw a complete diagram of this situation. Find and label each of the following: vertex, Horizontal Intercept (x-int) (positive side), and Vertical Intercept (y-int).

   b) How high above the ground was the bird when it was launched?

   c) After how many seconds does the bird reach its highest point?

   d) How high is the angry bird at its highest point?

   e) After how many seconds does the angry bird hit the ground?

   f) If the bird is traveling at 15 feet per second, how far does the angry bird travel before it hit the ground? *Use interval notation and include units.*

   g) Determine the practical domain and practical range of this function.

9. A company's revenue earned from selling x items is given by the function $R(x) = 680x$, and cost is given by $C(x) = 10000 + 2x^2$.

   a) Write a function, $P(x)$, that represents the company's *profit* from selling x items.

   b) Identify the Vertical Intercept (y-int) of $P(x)$. Write it as an ordered pair and interpret its meaning in a complete sentence.

   c) How many items must be sold in order to maximize the profit?

   d) What is the maximum profit?

   e) How many items does this company need to sell in order to break even?

   f) Determine the practical domain and practical range of this function. *Use interval notation and include units.*

10. An arrow is shot straight up into the air. The function $H(t) = -16t^2 + 90t + 6$ gives the height (in feet) of an arrow after *t* seconds. Round answers to two decimal places as needed. All answers must include appropriate units of measure.

    a) How long does it take for the arrow to reach its maximum height? Write your answer in a complete sentence.

    b) Determine the maximum height of the arrow. Write your answer in a complete sentence.

    c) How long does it take for the arrow to hit the ground? Write your answer in a complete sentence.

    d) Identify the Vertical Intercept (y-int). Write it as an ordered pair and interpret its meaning in a complete sentence.

    e) Determine the practical domain of *H*(*t*). *Use interval notation and include units.*

    f) Determine the practical range of *H*(*t*). *Use interval notation and include units.*

g) Use your graphing calculator to create a good graph of H(t). Use the practical domain and range to determine an appropriate viewing window. In the space below, sketch what you see on your calculator screen, and indicate the window you used.

Xmin:_________

Xmax:_________

Ymin:_________

Ymax:_________

h) Determine $H(3)$. Write a sentence explaining the meaning of your answer in the context of the arrow.

i) Use your graphing calculator to solve the equation $H(t) = 80$. Write a sentence explaining the meaning of your answer in the context of the arrow.

Section 5.4: Standard (Vertex) Form of Quadratic Functions

11. For each of the following functions,
A) Use your graphing calculator to find the vertex, equation of the axis of symmetry, horizontal intercept(s), vertical intercept, domain, and range of the function:
B) Draw a graph of the function labeling the Vertex, Axis of Symmetry, and two other points.

$f(x) = -(x-2)^2 + 4$

A)
Vertex:____________

Equation of the Axis of Symmetry:_____________

Domain:____________

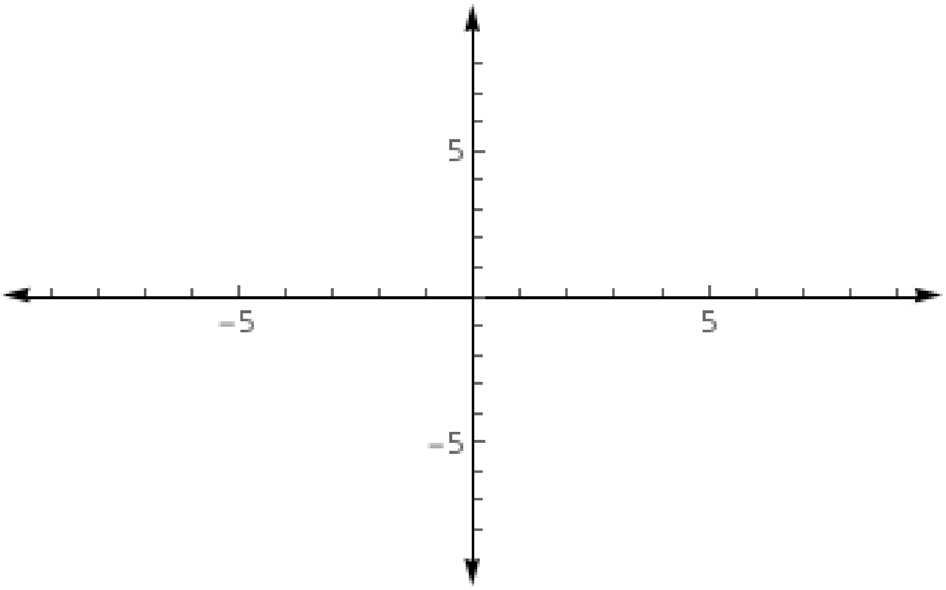
Since a=_____ Circle One: Opens Up or Opens Down

Circle One: Maximum or Minimum

Range:_____________

Horizontal Intercept(s) (Intersect Method):

Vertical Intercept(y int):
B)



12. $f\left(x\right)=\left(x+2\right)^2+2$

A)

Vertex:____________

Equation of the Axis of Symmetry:_____________

Domain:____________

Since a=_____ Circle One: Opens Up or Opens Down

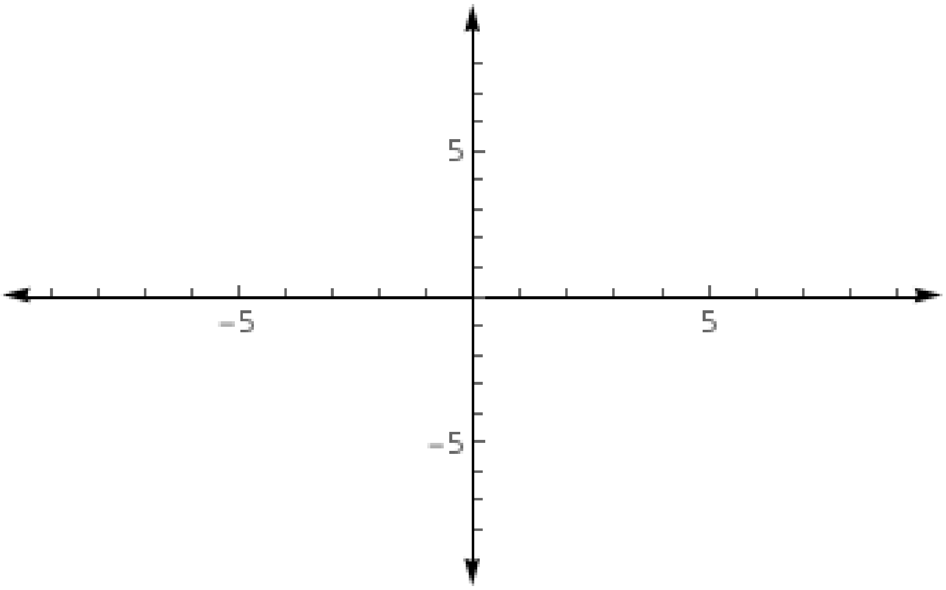
Circle One: Maximum or Minimum

Range:_____________

Horizontal Intercept(s) (Intersect Method):

Vertical Intercept(y-int):

B)



13. $f(x) = -2(x-2)^2 + 4$

A)

Vertex:____________

Equation of the Axis of Symmetry:_____________

Domain:____________

Since a=_____ Circle One: Opens Up or Opens Down

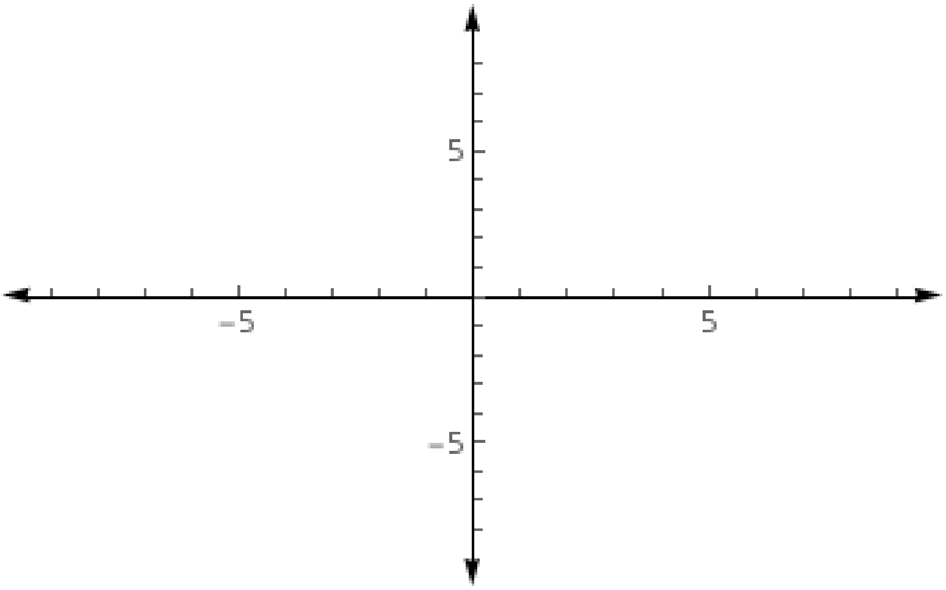
Circle One: Maximum or Minimum

Range:_____________

Horizontal Intercept(s) (Intersect Method):

Vertical Intercept(y-int):

B)



Section 5.5: Quadratic Regression

14. Fireworks were shot from a launching tower at an initial velocity of 70 feet per second. The data below show the height of the fireworks for varying amounts of time (in seconds).

t	1	2	3	4
$F(t)$	93	118	103	65

a) Use the Quadratic Regression feature of your calculator to generate a mathematical model for this situation.

b) Based on your model what is the height of the launching tower? Explain by writing a complete sentence.

c) Use your model to predict the height of the fireworks at 3 seconds. How does this compare to the value in the data table?

d) Using your model, for what values of t is the fireworks 75 feet high?

e) Use your model to determine how long it will take for the fireworks to hit the ground.

f) Use your model and the calculator to determine the maximum height.

g) Use your model and the calculator to determine the practical domain and practical range for this scenario. *Use interval notation and include units.*

h) Use your graphing calculator to create a good graph of *F(t)*. Use the practical domain and range to determine an appropriate viewing window. In the space below, sketch what you see on your calculator screen, and indicate the window you used.

Xmin:_________

Xmax:_________

Ymin:_________

Ymax:_________

15. Jupiter is the most massive planet in our solar system. Its gravity is 76 feet per second squared compared to Earth's 32 feet per second squared. The data below represent the height of a rocket launched from a hill on Jupiter.

t	1	2	3	4	5
$J(t)$	290	470	590	620	575

a) Use the Quadratic Regression feature of your calculator to generate a mathematical model for this situation. Round to three decimals.

b) Based on your model how high is the hill from which the rocket was launched ? Explain.

c) Use your model to predict the height of the rocket at 3 seconds. How does this compare to the value in the data table? Write a complete sentence to explain your answer.

d) Using your model, for what values of t is the rocket 450 feet high?

e) Use your model to determine how long it will take for the rocket to hit the surface of Jupiter (include units).

f) Use your model and the calculator to determine the maximum height.

g) Use your model to determine the practical domain and practical range for this scenario *Use interval notation and include units.*

h) Use your graphing calculator to create a good graph of *J(t)*. Use the practical domain and range to determine an appropriate viewing window. In the space below, sketch what you see on your calculator screen, and indicate the window you used.

Xmin:_________

Xmax:_________

Ymin:_________

Ymax:_________

16. A train tunnel is modeled by the quadratic function $h(x) = -0.45x^2 + 28.8$, where x is the distance, in feet, from the center of the tracks and $h(x)$ is the height of the tunnel, also in feet. Assume that the high point of the tunnel is directly in line with the center of the train tracks.

    a) Draw a complete diagram of this situation. Using the calculator, as needed, to find and label each of the following: vertex, Horizontal Intercept (x-int)(s) and Vertical Intercept (y-int). Round answers to the nearest tenth as needed.

    b) How high is the top of the tunnel?

    c) How wide is the base of the tunnel?

    d) A train with a flatbed car 6 feet off the ground is carrying a large object that is 12 feet high. How much room will there be between the top of the object and the top of the tunnel?

17. A company's profit, P, earned from selling x items is given by the table below.

x	10	80	150	225	300	340
P(x)	-3408	31622	47027	41751	13986	-9781

a) Use the Quadratic Regression feature of your calculator to write a function, $P(x)$, that represents the company's profit from selling x items. Use function notation and the appropriate variables. Round to two decimal places.

b) Use your graphing calculator to generate a scatterplot of the data *and* regression line on the same screen. You must use an appropriate viewing window. In the space below, draw what you see on your calculator screen, and write down the viewing window you used.

Xmin=____________

Xmax=____________

Ymin=____________

Ymax=___________

c) Using your function from part a), identify the Vertical Intercept (y-int) of $P(x)$. Write it as an ordered pair and interpret its meaning in a complete sentence. Round to the nearest item and the nearest cent.

d) Use the calculator to identify the vertex of the function found in part a) and interpret its meaning in a complete sentence. Round to the nearest item and the nearest cent.

e) How many items does this company need to sell in order to break even? Write your answer in a complete sentence. Round UP to the nearest item.

Lesson 5 Assessment

1. Fill out the following table. Intercepts must be written as ordered pairs. Always use proper notation. Round to two decimal places.

	$f(x)=2x^2-4x-30$	$g(x)=5-x^2$	$y=5x^2-4x+17$	$f(x)=-3(x+3)^2+6$
Opens Up or Down? Max or Min?				
Vertical Intercept (y-int)				
Horizontal Intercept (x-int)(s)				
Axis of Symmetry (Equation)				
Vertex				
Domain				
Range				

2. The function $H(t)=-16t^2+88t$ gives the height (in feet) of golf ball after *t* seconds. Round answers to two decimal places as needed. All answers must include appropriate units of measure.

   a) Determine the maximum height of the golf ball. Show your work. Write your answer in a complete sentence.

   b) How long does it take for the ball to hit the ground? Show your work. Write your answer in a complete sentence.

   c) Identify the Vertical Intercept (y-int). Write it as an ordered pair and interpret its meaning in a complete sentence.

   d) Determine the practical domain of *H*(*t*). *Use interval notation and include units.*

   e) Determine the practical range of *H*(*t*). *Use interval notation and include units.*

Lesson 6 – Solving Quadratic Equations

We will continue our work with Quadratic Functions in this lesson and will learn several methods for solving quadratic equations.

The Square Root Property is the first method you will work with to solve quadratic equations followed by factoring and then the quadratic formula. You will get a tiny taste of something called Complex Numbers and then will finish up by putting all the solution methods together.

Pay special attention to the problems you are working with and details such as signs and coefficients of variable terms. Extra attention to detail will pay off in this lesson.

Lesson Topics

Mini-Lesson 6

Section 6.1 – Standard (Vertex) Form of Quadratic Functions – Algebraic Approach

There are three main algebraic methods to solve quadratic equations we will explore:

1) Square Root Property (Quadratic Function is in standard form $f(x) = a(x-h)^2 + k$)

2) Solve by Factoring (Quadratic Function is in general form $f(x) = ax^2 + bx + c$)

3) Quadratic Formula (Quadratic Function is in general form $f(x) = ax^2 + bx + c$)

In additional studies, not covered in the workbook, you may also use the method of completing the square to solve a quadratic equation.

Standard (Vertex) Form of Quadratic Functions

$$f(x) = a(x-h)^2 + k \ , \ a \neq 0$$

where

$$Vertex = (h,k)$$

Equation of the Axis of Symmetry is $x = h$.

The Square Root property is used in this section to help solve, algebraically, for the **horizontal intercepts** of a quadratic function in standard form.

Square Root Property

(x is a real number)

$$x^2 = a$$

$$\sqrt{x^2} = \sqrt{a}$$

$$x = \pm\sqrt{a}$$

$$x = \sqrt{a} \text{ or } x = -\sqrt{a}$$

Problem 1	**WORKED EXAMPLE – Standard Form of Quadratic Functions**

A) Find the vertex, equation of the axis of symmetry, horizontal intercept(s), vertical intercept, domain, and range of the function $f(x) = -2(x-3)^2 + 1$

Solution:

$Vertex = (h,k) = (3,1)$

Equation of the Axis of Symmetry: $x = 3$

$Domain = (-\infty, \infty)$

Since a=__-2___ Circle One: Opens Up or [✓ Opens Down]

Circle One: [✓ Max] or Minimum

$Range = (-\infty, 1]$

Horizontal Intercepts (x-ints): (**Square Root Property**)

$$f(x) = -2(x-3)^2 + 1 = 0$$

Solve

$-2(x-3)^2 + 1 = 0$ Isolate the perfect square factor

$$-2(x-3)^2 = -1$$

$$(x-3)^2 = \frac{1}{2}$$

$$\sqrt{(x-3)^2} = \sqrt{\frac{1}{2}}$$

$$x - 3 = \pm\sqrt{\frac{1}{2}}$$

WARNING: Taking the square root of the left hand side of the above equation will yield TWO equations to solve by the square root property, namely:

$x - 3 = -\frac{1}{\sqrt{2}}$ and $x - 3 = \frac{1}{\sqrt{2}}$

$x = -\frac{1}{\sqrt{2}} + 3$ $x = \frac{1}{\sqrt{2}} + 3$

$x \approx 2.29$ $x \approx 3.71$

The **EXACT** form of the horizontal intercepts are the ordered pairs $\left(\frac{1}{\sqrt{2}} + 3, 0\right), \left(-\frac{1}{\sqrt{2}} + 3, 0\right)$.

The **APPROXIMATE** form of the horizontal intercepts are the ordered pairs $(2.29, 0), (3.71, 0)$

.

Vertical Intercept (y-int):

$f(0) = -2(0-3)^2 + 1$

$f(0) = -2(-3)^2 + 1$

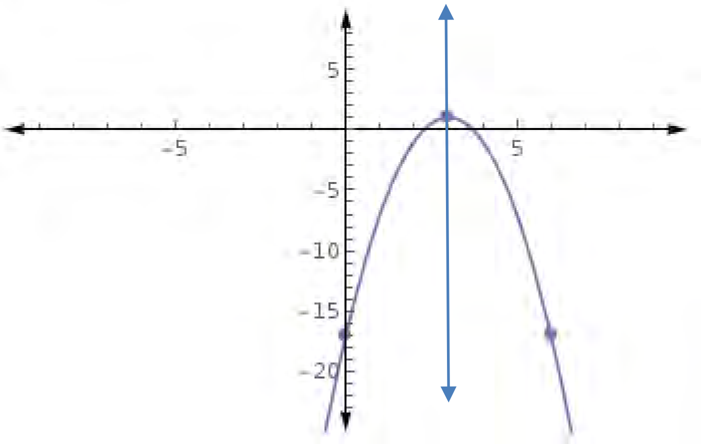
$f(0) = -2(9) + 1$

$f(0) = -18 + 1$

$f(0) = -17$

Vertical Intercept is the ordered pair $(0, -17)$.

B) Draw a graph of the function labeling the Vertex, Axis of Symmetry, and two other points.



Problem 2	**MEDIA EXAMPLE – Standard Form of Quadratic Functions**

A) Find the vertex, equation of the axis of symmetry, horizontal intercept(s), vertical intercept, domain, and range of the function $f(x) = (x-3)^2 - 4$

Vertex:____________

Equation of the Axis of Symmetry:_____________

Domain:____________

Since a=_____ Circle One: Opens Up or Opens Down

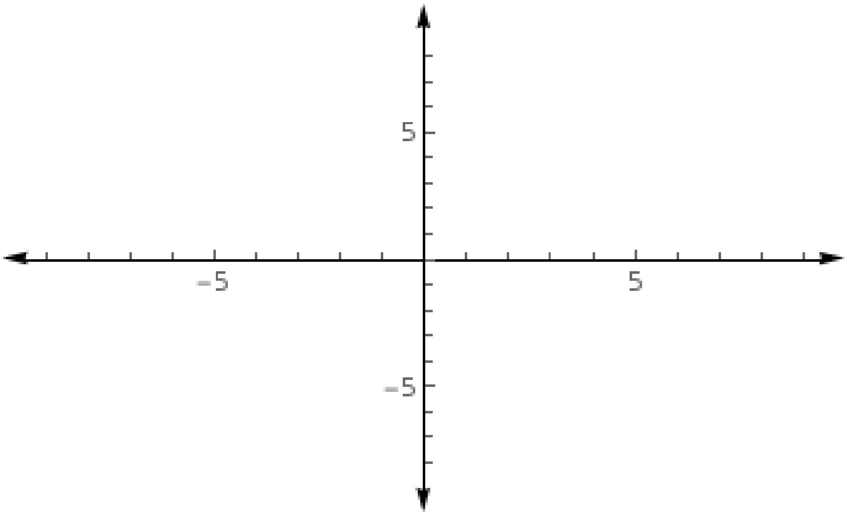
Circle One: Maximum or Minimum

Range:_____________

Horizontal Intercept(s) (x-int) (Square Root Property):

Vertical Intercept (y-int):

B) Draw a graph of the function labeling the Vertex, Axis of Symmetry, and two other points.



Problem 3	**YOU TRY – Standard Form of Quadratic Functions**

A) Find the vertex, equation of the axis of symmetry, horizontal intercept(s), vertical intercept, domain, and range of the function $f(x)=2(x-1)^2-4$

Vertex:____________

Equation of the Axis of Symmetry:_____________

Domain:____________

Since a=_____ Circle One: Opens Up or Opens Down

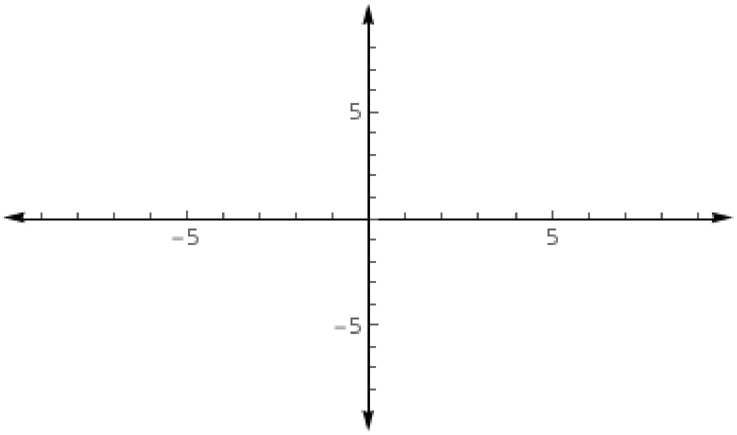
Circle One: Maximum or Minimum

Range:_____________

Horizontal Intercept(s) (x-int) (Square Root Property):

Vertical Intercept (y-int):

B) Draw a graph of the function labeling the Vertex, Axis of Symmetry, and two other points.



Section 6.2 – Factoring Quadratic Expressions ($ax^2 + bx + c, a = 1$)

So far, we have only used our graphing calculators to solve quadratic equations in general form utilizing the Intersection method. There are other methods to solve quadratic equations. The first method we will discuss is the method of FACTORING. Before we jump into this process, you need to have some concept of what it means to FACTOR using numbers that are more familiar.

Factoring Whole Numbers

To FACTOR the number 60, write all of the whole numbers that divide 60 with a zero remainder (we call this dividing a number evenly), these numbers are called FACTORS:

- $60 \div 1 = 60$ (not very interesting but true)
- $60 \div 2 = 30$
- $60 \div 3 = 20$
- $60 \div 4 = 15$
- $60 \div 5 = 12$
- $60 \div 6 = 10$

The set of all factors of 60 is $\{1,2,3,4,5,6,10,12,15,20,30,60\}$

Some factorizations of 60 are

$1 \cdot 60$

$2 \cdot 30$

$3 \cdot 20$

This means we wrote 60 as a product of some of the numbers that divide it evenly.
The most basic factorization of 60 is as a product of its prime factors (remember that prime numbers are only divisible by themselves and 1). The PRIME FACTORIZATION of 60 is:

$$60 = 2 \cdot 2 \cdot 3 \cdot 5$$

There is only one PRIME FACTORIZATION of 60 so we can now say that 60 is COMPLETELY FACTORED when we write it as $60 = 2 \cdot 2 \cdot 3 \cdot 5$.

When we factor polynomial expressions, we use a similar process. For example, to factor the expression $24x^2$, we would first find the prime factorization of 24 and then factor x^2.

$$24 = 2 \cdot 2 \cdot 2 \cdot 3 \quad \text{and} \quad x^2 = x \cdot x$$

Putting these factorizations together, we obtain the following:

$$24x^2 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x$$

Let's see how the information above helps us to factor more complicated polynomial expressions and ultimately leads us to a second solution method for quadratic equations.

Factoring the GCF (Greatest Common Factor) of an Expression

Problem 4	**WORKED EXAMPLE – Factoring Using GCF Method**

Factor $3x^2 + 6x$. Write your answer in completely factored form.

The building blocks of $3x^2 + 6x$ are the terms $3x^2$ and $6x$.

Step 1: Find the GCF (Greatest Common Factor) –

Factor $3x^2$: $3x^2 = 3 \cdot x \cdot x$

Factor $6x$: $6x = 2 \cdot 3 \cdot x$

The greatest factor in common between them is 3 (between 3 and 6) and x, so the GCF is 3x.

Step 2: Divide $3x^2 + 6x$ by the GCF, 3x.

$$\frac{3x^2 + 6x}{3x} = \frac{3x^2}{3x} + \frac{6x}{3x} = x + 2$$ (this is called the **quotient**)

Step 3: Re-write the original expression as the GCF multiplied by the quotient found in step 2:

$$3x^2 + 6x = GCF\left(Quotient\right) = 3x\left(x+2\right)$$

Step 4: CHECK!!!!!!!!!!

Distribute to check your work.

$$3x\left(x+2\right) = 3x \cdot \left(x\right) + 3x \cdot \left(2\right) = 3x^2 + 6x$$

Problem 5	**MEDIA EXAMPLE – Factoring Using GCF Method**

Factor the following quadratic expressions. Write your answers in completely factored form.

a) $11a^2 - 4a$

b) $55w^2 + 5w$

Problem 6	**YOU TRY – Factoring Using GCF Method**

Factor the following quadratic expression. Write your answers in completely factored form.

a) $64b^2 - 16b$

b) $11c^2 + 7c$

Factoring a Binomial from an Expression

Problem 7	**WORKED EXAMPLE – Factoring a Binomial Factor (Used in Factor by Grouping Method– Section 6.3)**

Factor the common binomial factor from the expression: $x(x+5)-2(x+5)$

Solution:

Step 1: Find the GCF: GCF=(x+5)

Since (x+5) is the greatest common (binomial) factor.

Step 2: Divide $x(x+5)-2(x+5)$ by the GCF, (x+5).

$$\frac{x(x+5)-2(x+5)}{(x+5)}=\frac{x(x+5)}{(x+5)}-\frac{2(x+5)}{(x+5)}=x-2$$ (this is called the **quotient**)

Step 3: Re-write the original expression as the GCF multiplied by the quotient found in step 2:

$$x(x+5)-2(x+5)=(GCF)(Quotient)=(x+5)(x-2)$$

$$x(x+5)-2(x+5)=(x+5)(x-2)$$

Step 4: CHECK!!!!!!!!!!!!

FOIL and distribute to check.

Problem 8	**YOU TRY – Factoring a Binomial Factor (Used in Factor by Grouping Method– Section 6.3)**

Factor the common binomial factor from the expression: $3x(2x-1)-(2x-1)$

Note: A method used to factor a quadratic expressions where **a=1** is shown below. The GCF of the examples below is 1.

Factoring Quadratic Expressions of the form $x^2 + bx + c$ by **TRIAL AND ERROR**

$$x^2 + bx + c = (x + p)(x + q),$$

where $b = p + q$ and $c = p \cdot q$

Problem 9	**WORKED EXAMPLE – Factoring Using Trial and Error**

Factor the quadratic expression $x^2 + 5x – 6$. Write your answer in completely factored form.

Step 1: Look to see if there is a common factor other than 1 in this expression. If there is, then you can use the GCF method to factor out the common factor.

The expression $x^2 + 5x – 6$ has no common factors other than 1.

Step 2: For this problem, $b = 5$ and $c = –6$. We need to identify p and q. In this case, these will be two numbers whose product is –6 and sum is 5. One way to do this is to *list* different numbers whose product is –6, then see which pair has a sum of 5.

Product = –6	Sum = 5
$–1 \cdot 6$	YES
$1 \cdot –6$	No
$–2 \cdot 3$	No
$-3 \cdot 2$	No

Step 3: Write in factored form

$$x^2 + 5x – 6 = (x + (–1))(x + 6)$$

$$x^2 + 5x – 6 = (x – 1)(x + 6)$$

Step 4: Check by foiling.

$$\begin{aligned}(x – 1)(x + 6) &= x^2 + 6x – x – 6 \\ &= x^2 + 5x – 6 \quad \text{CHECKS!}\end{aligned}$$

Problem 10	**MEDIA EXAMPLE – Factoring Using Trial and Error**

Factor each of the following quadratic expressions. Write your answers in completely factored form. Check your answers.

a) $a^2 + 7a + 12$ b) $w^2 + w - 20$ c) $x^2 - 36$

Problem 11	**YOU TRY – Factoring Using Trial and Error**

Factor each of the following quadratic expressions completely. Write your answers in completely factored form. Check your answers.

a) $n^2 + 8n + 7$ b) $r^2 + 3r - 70$ c) $m^2 - m - 30$

d) $3x^2 + 15x + 18$ e) $x^3 - x^2 - 12x$ f) $2x^3 + 16x^2 + 30x$

Section 6.3 – Factoring Difference of Two Squares (DOTS) Binomial, Factor by Grouping, and the AC Method for $ax^2+bx+c, a \neq 0,1$

After factoring out the GCF (Greatest Common Factor) of an algebraic expression, if it is other than 1, follow the steps below in order to factor a specific algebraic expression. For the purpose of simplicity, we are only considering examples that factor over the integers or are then prime.

You may refer to the **Factoring Map** in the **APPENDIX** to guide you through factoring.

Factoring a Difference of Two Squares (DOTS) Binomial	**Sum of Squares**
$a^2-b^2=(a+b)(a-b)$	$a^2+b^2=prime$

Problem 12	**WORKED EXAMPLE - Factoring a DOTS Binomial**

Factor: x^2-4y^2

Step 1: Rewrite each term as a perfect square to find a and b.

$$x^2-4y^2=x^2-(2y)^2$$

a is x
b is 2y

Step 2: Factor the binomial using the equation above: $a^2-b^2=(a+b)(a-b)$

$$x^2-4y^2=x^2-(2y)^2=(x-2y)(x+2y)$$

This is the factored form of the binomial. You may distribute (FOIL) to check that the factorization is correct.

Problem 13	**YOU TRY - Factoring a DOTS Binomial**

a) Factor Completely: x^2-81

b) x^2-1

c) $4x^2-9$

d) $3x^2+27$

Problem 14	**MEDIA EXAMPLE - Factoring a DOTS Binomial**

Factor Completely: $4x^2-36$

Factoring a Four-Term Polynomial: Factor by Grouping method

$$aX + bX + aY + bY = X(a+b) + Y(a+b) = (X+Y)(a+b)$$

Where a,b are variable expressions and X and Y represents the GCF's of the first two terms and second two terms respectively.

Problem 15	WORKED EXAMPLE - Factoring a Four-Term Polynomial

If we have a four term polynomial, factor: $2x^2 - 10x + x - 5$

Step 1: Find the GCF (Greatest Common Factor) of the first two terms and the GCF of the second two terms of the polynomial.

In this case the GCF of $2x^2 - 10x$ is 2x and the GCF of $x - 5$ is 1.

Step 2: Factor out the respective GCF's from the two binomials.

$$2x^2 - 10x + x - 5 = 2x(x-5) + 1(x-5)$$

Step 3: Factor out the common binomial factor (the factor that looks exactly the same) of the expression (refer to Problem 7).

$$2x^2 - 10x + x - 5 = 2x(x-5) + 1(x-5) = (x-5)(2x+1)$$

This is the factored form of the four-term polynomial. You may distribute (FOIL) to check that the factorization is correct.

Problem 16	YOU TRY - Factoring a Four-Term Polynomial

Factor: $3x^2 + 2x + 12x + 8$

Factoring a Trinomial ax^2+bx+c where $a\neq 0,1$:AC method

Idea of the AC method: To write a trinomial with the conditions above as a four-term polynomial in order to perform factor by grouping.

Problem 17	**WORKED EXAMPLE - Factoring a Trinomial ax^2+bx+c where $a\neq 0,1$: AC method**

Factor: $2x^2+9x+4$

Step 1: Multiply a and c together to find **ac**.

a=2, c=4

ac=2*4=8

Step 2: Find two numbers that multiply to **ac** and add to b.

1*8=8=**ac**

1+8=9=b

Step 3: Rewrite the term bx as two terms using the factors of **ac, in this case 8 and 1,** as the coefficients of x, found in step 2.

$$2x^2+9x+4=2x^2+8x+x+4$$

Step 4: Apply factor by grouping (refer to previous page) to the four-term polynomial.

$$2x^2+9x+4=2x^2+8x+x+4=2x(x+4)+1(x+4)=(x+4)(2x+1)$$

This is the factored form of the trinomial. You may distribute (FOIL) to check that the factorization is correct.

Problem 18	**MEDIA EXAMPLE - Factoring a Trinomial ax^2+bx+c where $a\neq 0,1$: AC method**

Factor: $3x^2-4x+1$

Problem 19	**MEDIA EXAMPLE - Factoring a Trinomial ax^2+bx+c where $a \neq 0,1$: AC method**

Factor: a) $4x^2-4x-15$ b) $20x^2+19x+3$

Problem 20	**YOU TRY - Factoring a Trinomial ax^2+bx+c where $a \neq 0,1$: AC method**

Factor: $2x^2+9x+7$

Problem 21	**YOU TRY - Factoring a Trinomial ax^2+bx+c where $a \neq 0,1$: AC method**

Factor: $3x^2+4x+1$

Section 6.4 – Solving Quadratic Equations by Factoring

In this section, we will see how a quadratic equation written in general form $ax^2 + bx + c = 0$ can be solved *algebraically* using FACTORING methods.

The Zero Product Principle
If $a \cdot b = 0$, then $a = 0$ or $b = 0$

To solve a Quadratic Equation by FACTORING:

Step 1: Make sure the quadratic equation is set equal to zero: $ax^2 + bx + c = 0$

Step 2: Write the left side in Completely Factored Form

Step 4: Apply the ZERO PRODUCT PRINCIPLE
Set each linear factor equal to 0 and solve for x

Step 5: Verify result by graphing and finding the intersection point(s).

Problem 22	**WORKED EXAMPLE–Solve Quadratic Equations By Factoring**

a) Solve by factoring: $5x^2 – 10x = 0$

Step 1: This quadratic equation is already set equal to zero.

Step 2: Check if there is a common factor, other than 1, for each term (yes…$5x$ is common to both terms)

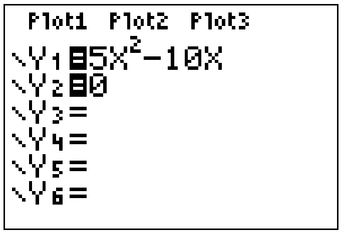
Step 3: Write the left side in Completely Factored Form

$$5x^2 - 10x = 0$$
$$5x(x - 2) = 0$$

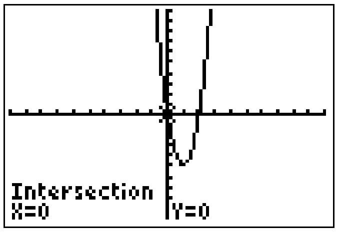
Step 4: Set each linear factor equal to 0 and solve for x:

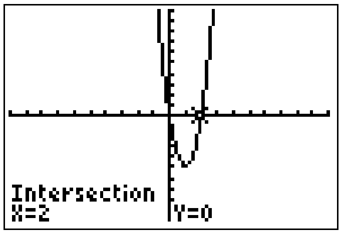
$$5x = 0 \quad \text{OR} \quad x - 2 = 0$$
$$x = 0 \quad \text{OR} \quad x = 2$$

Step 5: Verify result by graphing.



```
Plot1 Plot2 Plot3
\Y1=5X²-10X
\Y2=0
\Y3=
\Y4=
\Y5=
\Y6=
```





b) Solve by factoring: $x^2 - 7x + 12 = 2$

Step 1: Make sure the quadratic is set equal to zero.
Subtract 2 from both sides to get: $x^2 - 7x + 10 = 0$

Step 2: Check if there is a common factor, other than 1, for each term.
Here, there is no common factor.

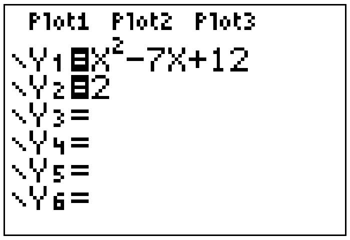
Step 3: Write the left side in Completely Factored Form

$$x^2 - 7x + 10 = 0$$
$$(x + (-5))(x + (-2)) = 0$$
$$(x - 5)(x - 2) = 0$$

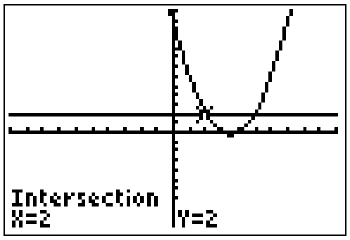
Step 4: Set each linear factor to 0 and solve for x:

$$(x - 5) = 0 \quad \text{OR} \quad (x - 2)= 0$$
$$x = 5 \quad \text{OR} \quad x = 2$$

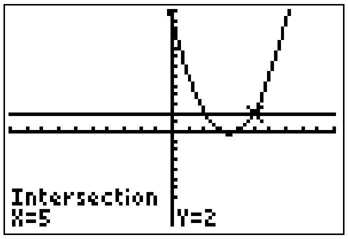
Step 5: Verify result by graphing (Let Y1 = $x^2 - 7x + 12$, Y2 = 2)



```
 Plot1 Plot2 Plot3
\Y1■X²-7X+12
\Y2■2
\Y3=
\Y4=
\Y5=
\Y6=
```



Intersection
X=2 Y=2



Intersection
X=5 Y=2

Problem 23	**MEDIA EXAMPLE–Solve Quadratic Equations By Factoring**

Solve the equations below by factoring. Show all of your work. Verify your result by graphing.

a) Solve by factoring: $-2x^2 = 8x$

b) Solve by factoring: $x^2 = 3x + 28$

c) Solve by factoring: $x^2 + 5x = x - 3$

Problem 24 | YOU TRY – Solving Quadratic Equations by Factoring

Use an appropriate factoring method to solve each of the quadratic equations below. Show all of your work. Be sure to write your final solutions using proper notation. Verify your answer by graphing. Sketch the graph on a good viewing window (the vertex, Vertical Intercept (y-int), and any Horizontal Intercept (x-int)s should appear on the screen). **Mark and label** the solutions on your graph.

a) Solve $x^2 + 3x = 10$

b) Solve $3x^2 = 17x$

c)Solve by factoring: $x^2 - 5 = 20$

Section 6.5 – The Quadratic Formula

You may refer to the **APPENDIX** at this point to review simplifying square roots.

The Quadratic Formula can be used to solve quadratic equations written in general form:

$$ax^2 + bx + c = 0 \text{ , } a \neq 0$$

The Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

To solve a Quadratic Equation using the QUADRATIC FORMULA:

Step 1: Make sure the quadratic equation is set equal to zero: $ax^2 + bx + c = 0$
Step 2: Identify the coefficients a, b, and c.
Step 4: Substitute these values into the Quadratic Formula
Step 5: Simplify your result completely.
Step 6: Verify your result by graphing and finding the horizontal intercept(s).

You can derive this formula directly from the quadratic equation in general form. $ax^2 + bx + c = 0$ using a factoring method called COMPLETING THE SQUARE. You will not be asked to use COMPLETING THE SQUARE in this class, but go through the information below and try to follow each step.

How to Derive the Quadratic Formula From $ax^2 + bx + c = 0$, $a \neq 0$

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$ [Subtract c from both sides then divide all by a]

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$ [Take the coefficient of x, divide it by 2, square it, and add to both sides]

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$$ [Factor the left side. On the right side, get a common denominator of $4a^2$]

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$ [Combine the right side to one fraction then take square root of both sides]

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$ [Simplify the square roots]

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$ [Solve for x]

$$x = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$ [Combine to obtain the final form for the Quadratic Formula]

Problem 25 WORKED EXAMPLE– Solve Quadratic Equations Using the Quadratic Formula

Solve the quadratic equation by using the Quadratic Formula. Verify your result by graphing and finding the horizontal intercept(s).

Solve $3x^2 - 2 = -x$ using the quadratic formula.

Step 1: Make sure the quadratic equation is set equal to zero: $3x^2 + x - 2 = 0$

Step 2: Identify $a = 3$, $b = 1$, and $c = -2$

Step 3: $x = \dfrac{-(1) \pm \sqrt{(1)^2 - 4(3)(-2)}}{2(3)} = \dfrac{-1 \pm \sqrt{1-(-24)}}{6} = \dfrac{-1 \pm \sqrt{1+24}}{6} = \dfrac{-1 \pm \sqrt{25}}{6}$

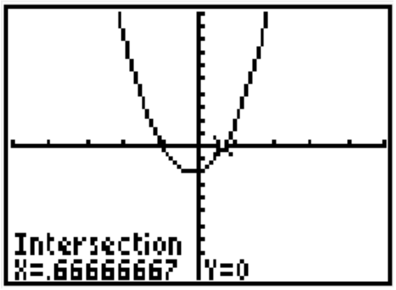
Step 4: Make computations for x_1 and x_2 as below and note the complete simplification process:

$$x_1 = \frac{-1+\sqrt{25}}{6} = \frac{-1+5}{6} = \frac{4}{6} = \frac{2}{3} \qquad x_2 = \frac{-1-\sqrt{25}}{6} = \frac{-1-5}{6} = -\frac{6}{6} = -1$$

Final solution $x = \dfrac{2}{3}$, $x = -1$

Step 5: Check by graphing.

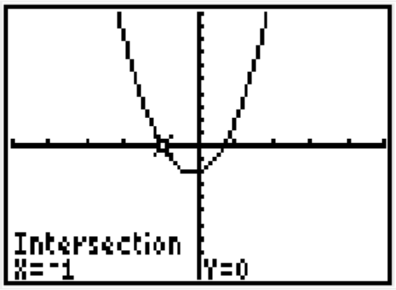
Graphical verification of Solution $x = \dfrac{2}{3}$



```
Intersection
X=.66666667  Y=0
```

[Note that $\frac{2}{3} \approx .6666667$]

Graphical verification of Solution $x = -1$



```
Intersection
X=-1         Y=0
```

You can see by the graphs above that this equation is an example of the "Case 2", in section 5.1, possibility of two, unique real number solutions for a given quadratic equation.

Problem 26 MEDIA EXAMPLE – Solve Quadratic Equations Using Quadratic Formula

Solve each quadratic equation by using the Quadratic Formula. Verify your result by graphing and finding the horizontal intercept(s).

$$\text{Quadratic Formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a) Solve $-x^2 + 3x + 10 = 0$

b) Solve $2x^2 - 4x = 3$

Problem 27 YOU TRY – Solve Quadratic Equations Using Quadratic Formula

Solve $3x^2 = 7x + 20$ using the Quadratic Formula. Show all steps and simplify your answer. Verify your answer by graphing. Sketch the graph on a good viewing window (the vertex, Vertical Intercept (y-int), and any Horizontal Intercepts (x-int) should appear on the screen). **Mark and label** the solutions on your graph.

Section 6.6 – Complex Numbers

Suppose we are asked to solve the quadratic equation $x^2 = -1$. Well, right away you should think that this looks a little weird. If I take any real number, other than zero, times itself, the result is always positive. Therefore, there is no REAL number x such that $x^2 = -1$. [Note: See explanation of Number Systems on the next page]

Hmmm…well, let's approach this using the Quadratic Formula and see what happens.

To solve $x^2 = -1$, need to write in standard form as $x^2 + 1 = 0$. Thus, $a = 1$ and $b = 0$ and $c = 1$.

Plugging these in to the quadratic formula, I get the following:

$$x = \frac{-0 \pm \sqrt{0^2 - 4(1)(1)}}{2(1)} = \frac{\pm\sqrt{-4}}{2} = \frac{\pm\sqrt{4(-1)}}{2} = \frac{\pm\sqrt{4}\sqrt{-1}}{2} = \frac{\pm 2\sqrt{-1}}{2} = \pm\sqrt{-1}$$

Well, again, the number $\sqrt{-1}$ does not live in the real number system nor does the number $-\sqrt{-1}$ yet these are the two solutions to our equation $x^2 + 1 = 0$.

The way mathematicians have handled this problem is to define a number system that is an extension of the real number system. This system is called the Complex Number System and it has, as its base defining characteristic, that equations such as $x^2 + 1 = 0$ can be solved in this system. To do so, a special definition is used and that is the definition that:

$$i = \sqrt{-1}$$

With this definition, then, the solutions to $x^2 + 1 = 0$ are just $x = i$ and $x = -i$ which is a lot simpler than the notation with negative under the radical.

When Will We See These Kinds of Solutions?

We will see solutions that involve the complex number "i" when we solve quadratic equations that never cross the x-axis. You will see several examples to follow that will help you get a feel for these kinds of problems.

Complex Numbers *a* + *bi*

Complex numbers are an extension of the real number system.
Standard form for a complex number is

$$a + bi$$

where a and b are real numbers,

$$i = \sqrt{-1}$$

Problem 28 | **WORKED EXAMPLE – Complex Numbers**

a) $\sqrt{-9} = \sqrt{9}\sqrt{-1}$

$= 3\sqrt{-1}$

$= 3i$

b) $\sqrt{-7} = \sqrt{7}\sqrt{-1}$

$= \sqrt{7}i$ or $i\sqrt{7}$

c) $\dfrac{3+\sqrt{-49}}{2} = \dfrac{3+\sqrt{49}\sqrt{-1}}{2}$

$= \dfrac{3+7i}{2}$

$= \dfrac{3}{2} + \dfrac{7}{2}i$

THE COMPLEX NUMBER SYSTEM

Complex Numbers:

All numbers of the form $a + bi$ where a, b are real numbers and $i = \sqrt{-1}$
Examples: $3 + 4i$, $2 + (-3)i$, $0 + 2i$, $3 + 0i$

Real Numbers – all the numbers on the REAL NUMBER LINE – include all RATIONAL NUMBERS and IRRATIONAL NUMBERS

Rational Numbers :

- ratios of integers
- decimals that terminate or repeat
- Examples:

$0.50 = \frac{1}{2}$, $-.75 = -\frac{3}{4}\text{-}\frac{3}{4}$,

$0.43 = \frac{43}{100}\frac{43}{137}$, $0.33 = \frac{33}{100}$

Integers: Zero, Counting Numbers and their negatives
{…–5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, …}

Whole Numbers: Counting Numbers and Zero
{0, 1, 2, 3, 4, 5, 6, 7, …..}

Counting Numbers
{1, 2, 3, 4, 5, 6, 7, …..}

Irrational Numbers

Examples: π, e, $\sqrt{5}$, $\sqrt{47}$ $\sqrt{11}$

- Decimal representations for these numbers never terminate and never repeat

Complex numbers are an extension of the real number system. As such, we can perform operations on complex numbers. This includes addition, subtraction, multiplication, and powers. A complex number is written in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$

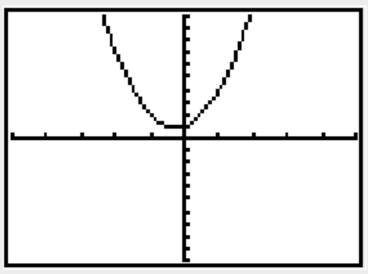
Section 6.7 – Complex Solutions to Quadratic Equations

Work through the following to see how to deal with equations that can only be solved in the Complex Number System.

Problem 29	**WORKED EXAMPLE – Solving Quadratic Equations with Complex Solutions**

Solve $2x^2 + x + 1 = 0$ for x. Leave results in the form of a complex number, $a+bi$.

First, graph the two equations as Y1 and Y2 in your calculator and view the number of times the graph crosses the x-axis. The graph below shows that the graph of $y = 2x^2 + x + 1$ does not cross the x-axis at all. This is an example of our Case 3, in section 5.1, possibility and will result in no Real Number solutions but two unique Complex Number Solutions.



To find the solutions, make sure the equation is set equal to zero (check).

Identify the coefficients $a = 2$, $b = 1$, $c = 1$.

Insert these into the quadratic formula and simplify as follows:

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(1)}}{2(2)} = \frac{-1 \pm \sqrt{1-8}}{4} = \frac{-1 \pm \sqrt{-7}}{4}$$

Break this into two solutions and use the a+bi form to get

$$\begin{aligned} x_1 &= \frac{-1+\sqrt{-7}}{4} \\ &= -\frac{1}{4} + \frac{\sqrt{-7}}{4} \\ &= -\frac{1}{4} + \frac{i\sqrt{7}}{4} \\ &= -\frac{1}{4} + \frac{\sqrt{7}}{4}i \end{aligned}$$

and

$$\begin{aligned} x_2 &= \frac{-1-\sqrt{-7}}{4} \\ &= -\frac{1}{4} - \frac{\sqrt{-7}}{4} \\ &= -\frac{1}{4} - \frac{i\sqrt{7}}{4} \\ &= -\frac{1}{4} - \frac{\sqrt{7}}{4}i \end{aligned}$$

The final solutions are $x_1 = -\frac{1}{4} + \frac{\sqrt{7}}{4}i,\ x_2 = -\frac{1}{4} - \frac{\sqrt{7}}{4}i$

Remember that $\sqrt{-1} = i$ so $\sqrt{-7} = i\sqrt{7}$

Work through the following problem to put the solution methods of graphing, factoring and quadratic formula together while working with the same equation.

Problem 30 | **YOU TRY – SOLVING QUADRATIC EQUATIONS**

Given the quadratic equation $x^2 + 3x - 7 = 3$, solve using the processes indicated below.

a) Solve by graphing (use your calculator and the Intersection process). Sketch the graph on a good viewing window (the vertex, Vertical Intercept (y-int), and any Horizontal Intercept (x-int)s should appear on the screen). **Mark and label** the solutions on your graph.

b) Solve by factoring. Show all steps. Clearly identify your final solutions.

c) Solve using the Quadratic Formula. Clearly identify your final solutions.

Lesson 6 Practice Problems

Round to two decimal places unless stated otherwise.

Section 6.1: Standard (Vertex) Form of Quadratic Functions – Algebraic Approach

For each of the following functions,
A) Find the vertex, equation of the axis of symmetry, horizontal intercept(s) (EXACT and APPROXIMATE form), vertical intercept, domain, and range of the function:
B) Draw a graph of the function labeling the Vertex, Axis of Symmetry, and two other points.

1) $f(x) = -(x+1)^2 + 3$

A)
Vertex:____________

Equation of the Axis of Symmetry:_____________

Domain:____________

Since a=_____ Circle One: Opens Up or Opens Down

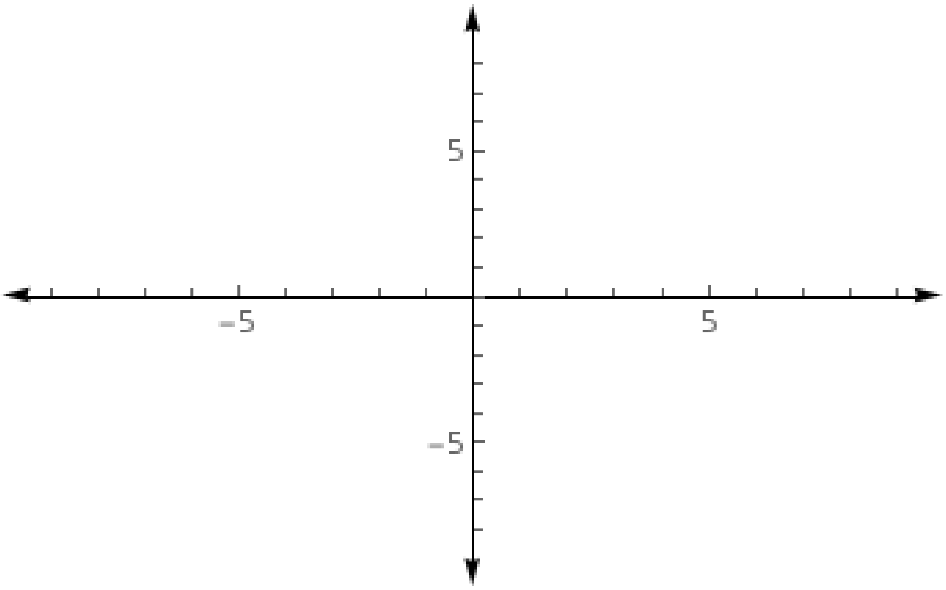
Circle One: Maximum or Minimum

Range:_____________

Horizontal Intercept(s) (x-int) (Square Root Property):

Vertical Intercept(y-int) (algebraically):

B)



2) $f(x) = (x-3)^2$

A)

Vertex:____________

Equation of the Axis of Symmetry:_____________

Domain:____________

Since a=_____ Circle One: Opens Up or Opens Down

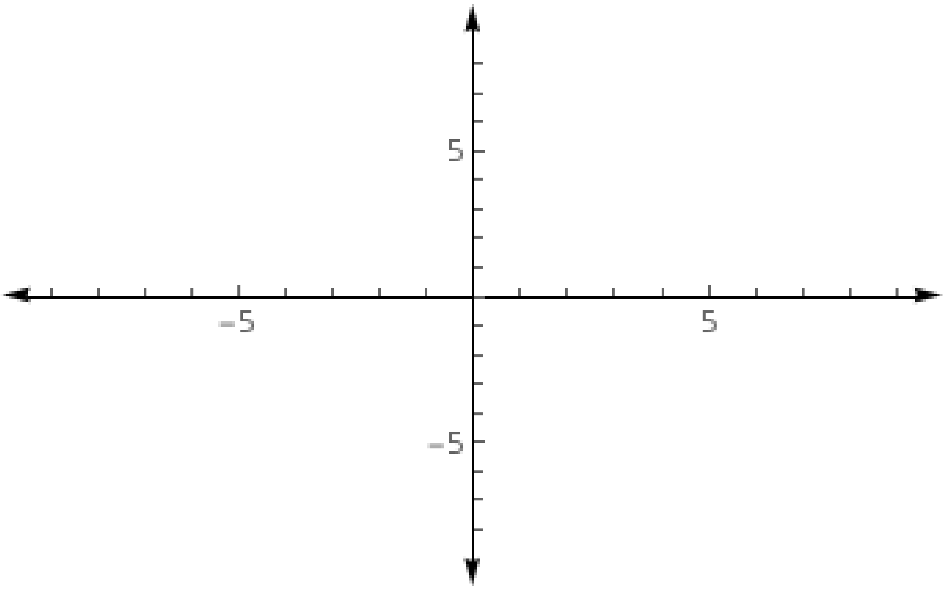
Circle One: Maximum or Minimum

Range:_____________

Horizontal Intercept(s) (x-int) (Square Root Property):

Vertical Intercept(y-int) (algebraically):

B)



3) $f(x)=(x+1)^2-5$

A)

Vertex:____________

Equation of the Axis of Symmetry:_____________

Domain:____________

Since a=_____ Circle One: Opens Up or Opens Down

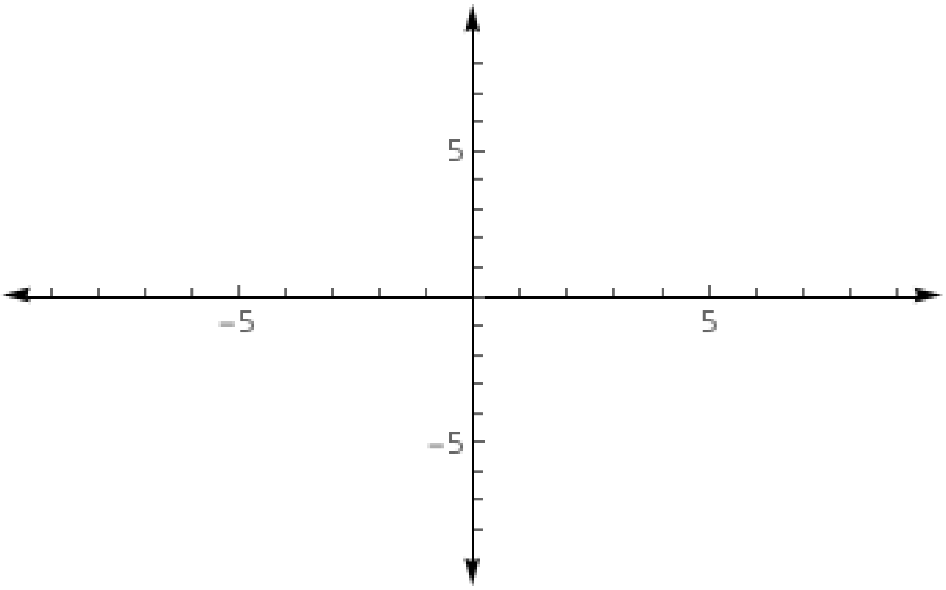
Circle One: Maximum or Minimum

Range:_____________

Horizontal Intercept(s) (x-int) (Square Root Property):

Vertical Intercept(y-int) (algebraically):

B)



4) The height of a rock thrown from a cliff and the height of the rock after t seconds is given by the function $H(t) = -16(t-3)^2 + 176$. If the rock reaches a maximum height of 176 feet off of the ground after 3 seconds then,

A) How long does it take for the rock to reach its maximum height and what is the maximum height of the rock?

B) After how many seconds does the rock hit the ground?

C) State the practical domain and practical range of $H(t)$ using interval notation (include units).

D) Sketch the graph of the function below labelling the vertex and two other points:

Xmin=______

Xmax=______

Ymin=______

Ymax=______

Section 6.2: Factoring Quadratic Expressions

5. Factor each of the following quadratic expressions. Write your answers in completely factored form.

a) x^2+7x+6

b) $3x^2+6x+3$

c) x^2+x-20

d) $x^2-12x+11$

e) x^2-5x+6

f) $3x^2+12x$

g) $20x^2-5x$

h) x^2+8x

i) $-3x(x+1)+2(x+1)$

j) $x(2x-1)-(2x-1)$

k) $5(y-3)+y(y-3)$

l) $3x(x-2)+2(x-2)$

Section 6.3: Factoring Difference of Two Squares (DOTS) Binomial, Factor by Grouping, and the AC Method

6. Factor Completely; if the expression is prime then indicate so:

1) $9b^2 - 16c^2$

2) $3x^2 - 2x + 5$

3) $7x^2 + 21x - x - 3$

4) $x^2 - 8x - 9$

5) $3x^2 - 18x + 4x - 24$

6) $5x^2 - 10x + 15$

7) $4x^2 - 40x$

8) $4x^2 + 25y^2$

9) $3x^2 - 6x + 24$

10) $x^2 - 36$

11) $2x^2 - 11x + 12$

12) $2t^2 - 288$

13) $2x^2 - x - 6$

14) $7x^2 - 8x + 1$

15) $30y^2 - 43y - 3$

Section 6.4: Solving Quadratic Equations by Factoring

7. Solve each of the following Quadratic Equations by Factoring (GCF). Be sure to write your final solutions using proper notation. Verify your answer by graphing. Sketch the graph on a good viewing window (the vertex, Vertical Intercept (y-int), and any Horizontal Intercept (x-int)s should appear on the screen). **Mark and label** the solutions on your graph.

   a) $4x^2 - 8x = 0$

   b) $9x^2 - 6x = 0$

   c) $2x^2 = 4x$

8. Use the Trial and Error Factoring Method to solve each of the quadratic equations below. Be sure to write your final solutions using proper notation. Verify your answer by graphing. Sketch the graph on a good viewing window (the vertex, Vertical Intercept (y-int), and any Horizontal Intercept (x-int(s)) should appear on the screen). **Mark and label** the solutions on your graph.

a) $x^2 + 8x + 12 = 0$

b) $x^2 - 42 = x$

c) $x^2 - 4x = 5$

d) $x^2 - 36 = 0$

9. Use an appropriate factoring method to solve each of the quadratic equations below. Be sure to write your final solutions using proper notation. Verify your answer by graphing. Sketch the graph on a good viewing window (the vertex, Vertical Intercept (y-int), and any Horizontal Intercept (x-int)s should appear on the screen). **Mark and label** the solutions on your graph.

   a) $9x^2 + 15x = 0$

   b) $x^2 + 10x - 24 = 0$

   c) $2x^2 - 4x - 30 = 0$

Section 6.5: The Quadratic Formula

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

10. Solve each quadratic equation by using the Quadratic Formula.
    - Make sure your equation is set equal to zero.
    - Identify the coefficients *a, b, c*
    - Substitute these values into the quadratic formula
    - Simplify your result completely then check your solution graphically
    - Mark and label the solutions on your graph. Be sure to include the exact and approximate solutions.

a) Solve $2x^2 - 2x - 4 = 0$ (This one is a fill in the blank)

$a =$ _____, $b =$ ______, $c =$ ______

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$$

$$x = \frac{(\quad) \pm \sqrt{(\quad) - (\quad)}}{(\quad)}$$

$$x = \frac{(\quad) \pm \sqrt{(\quad)}}{(\quad)}$$

$$x_1 = \frac{(\ )+\sqrt{(\ )}}{(\ )} \text{ and } x_2 = \frac{(\ )-\sqrt{(\ )}}{(\ )}$$

$$x_1 = \frac{(\ )+(\ )}{(\ )} \text{ and } x_2 = \frac{(\ )-(\ )}{(\ )}$$

$$x_1 = \frac{(\ )}{(\ )} \text{ and } x_2 = \frac{(\ )}{(\ )}$$

$$x_1 = 2 \text{ and } x_2 = -1$$

Final solution x = -1, 2 (Be sure to verify graphically)

Sketch the graph on a good viewing window (the vertex, Vertical Intercept (y-int), and any Horizontal Intercept (x-int)s should appear on the screen). **Mark and label** any real solutions on the graph.

Xmin = ________

Xmax = ________

Ymin = ________

Ymax = ________

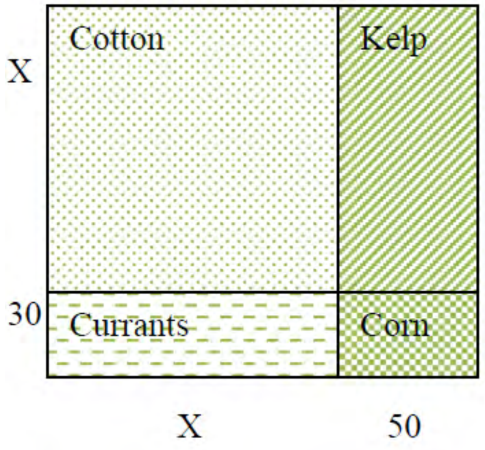
b) $2x^2 - 5x = 4$

c) $4x^2 - 2x = 6$

d) $6x^2 - 4x = 1$

e) $-2x^2 = 3x + 12$

11. Farmer Treeman wants to plant four crops on his land, Cotton, Corn, Kelp and Currants. He has 40,000 square feet for planting. He needs the length and width of the property to be as shown in the picture below (measured in feet). He determines the equation for the area of his property is $x^2 + 80x + 1500 = 40000$



a) What will the length and width of the property need to be? Show your work.

b) Determine the area of each section of the land. Include units in your answers.

Cotton: ____________________ Kelp:______________________

Currants: ____________________ Corn: ______________________

Name:________________________ **Date:**__________________

Lesson 6 Assessment

1. Factor each of the following quadratic expressions. Write your answers in factored form.

a) $x^2 - 6x + 8$

b) $3x^2 + 3x - 6$

c) $3x^2 + 11x + 6$

d) $x^2 - 8x + 16$

e) $15x^2 - 3x$

f) $x^2 - 9$

2. Solve the following equations *algebraically* (Factoring or Quadratic Formula or Square Root Property). You must show all algebraic steps for full credit. Where applicable, give both the exact answers and the decimal approximations rounded to three decimal places. Write complex solutions in the form $x = a + bi$ and $x = a - bi$ Use your calculator to *check* your answers. Sketch the graph on a good viewing window (the vertex, Vertical Intercept (y-int), and any Horizontal Intercepts (x-ints) should appear on the screen). **Mark and label** any real solutions on the graph.

a) $3x^2 + 2x + 3 = 8$

b) $x^2+9x+11=x-5$

c) $-2\left(x-2\right)^2=-8$

3. Solve the following equations by factoring:

a) $x^2-7x+10=0$

b) $2x^2-128=0$

c) $-4x^2+16x=0$

4. A) Find the vertex, equation of the axis of symmetry, horizontal intercept(s) (EXACT and APPROXIMATE form), vertical intercept, domain, and range of the function:
B) Draw a graph of the function labeling the Vertex, Axis of Symmetry, and two other points.

$f(x) = (x-1)^2 - 2$

A)
Vertex:____________

Equation of the Axis of Symmetry:_____________

Domain:____________
Since a=_____ Circle One: Opens Up or Opens Down

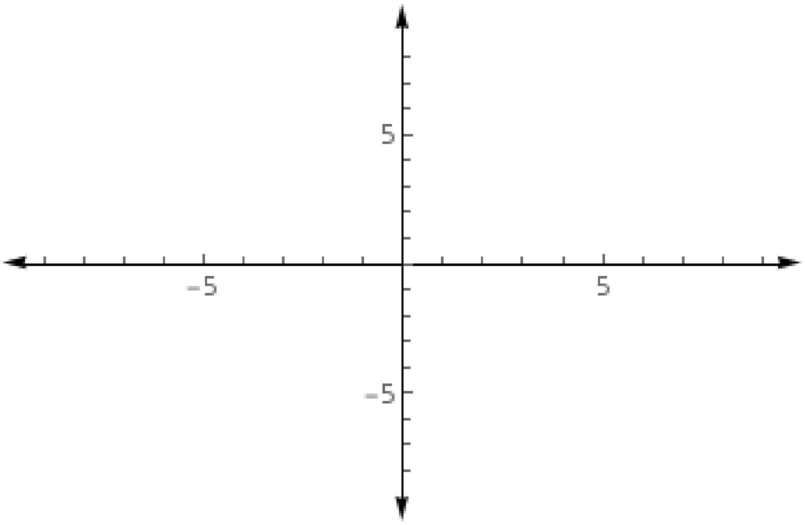
Circle One: Maximum or Minimum

Range:_____________

Horizontal Intercept(s) (x-int) (Square Root property):

Vertical Intercept(y-int) (algebraically):

B)



Lesson 7 – Radical Functions and Rational Exponents

In this lesson, you will embark on a study of radical functions. As the name implies, these functions contain at least one radical expression in them. Radical functions are mathematically connected to quadratic functions and that is why we study them after quadratics.

In this lesson, you will use two symbolic means to express and evaluate a root, graph and explore the key characteristics of various radical functions, solve radical equation both graphically and algebraically, and apply these techniques to model and solve applications involving radical functions.

Lesson Topics

Section 7.1 – Roots, Radicals, and Rational Exponents

SQUARE ROOTS

The square root of a is written as $\sqrt{a}$. If $\sqrt{a} = b$ then $b^2 = a$.

NOTATION:

The notation $\sqrt{a}$, is RADICAL NOTATION for the square root of a.

The notation $a^{\frac{1}{2}}$ is RATIONAL EXPONENT NOTATION for the square root of a.

On your TI 83/84 calculator, you can use the $\sqrt{}$ symbol to compute square roots.

EXAMPLES:

a) $\sqrt{25} = 5$ because $5^2 = 25$

b) Note that the square and square root "undo" each other:

$$\left(\sqrt{144}\right)^2 = 144 \text{ and } \sqrt{144^2} = 144$$

c) $25^{1/2} = \sqrt{25} = 5$

d) $\sqrt{-64} = (-64)^{1/2}$ is not a real number because there is no real number, squared, that will give -64

THE NTH ROOT

$\sqrt[n]{a} = a^{1/n}$, the nth root of a, n is a whole number greater than 0.

Notation for square root: By convention we write $\sqrt{a}$ instead of $\sqrt[2]{a}$.

EXAMPLES:

a) $\sqrt[4]{256} = 256^{1/4} = 4$	Calculator Entry: 256^(1/4)
b) $\sqrt[7]{-2187} = \left(-2187\right)^{1/7} = -3$	Calculator Entry: (-2187)^(1/7)
c) $-\sqrt[3]{15} = -15^{1/3} \approx -2.47$	Calculator Entry: -15^(1/3)
d) $\sqrt[6]{-324} = (-324)^{1/6}$ is not a real number	Calculator Entry: (-324)^(1/6)

RATIONAL EXPONENTS: Assume a is a positive real number and p, q are integers and q is greater than 0.

$$a^{p/q} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$$

EXAMPLES:

a) $\left(\sqrt[4]{81}\right)^3 = (81)^{3/4} = (3)^3 = 27$

b) $\left(\sqrt[3]{-125}\right)^2 = (-125)^{2/3} = (-5)^2 = 25$

c) $\dfrac{1}{\left(\sqrt[5]{32}\right)^2} = \dfrac{1}{32^{2/5}} = 32^{-2/5} = \dfrac{1}{2^2} = \dfrac{1}{4}$

Problem 1	**MEDIA EXAMPLE – Compute with Rational/Radical Exponents**

Compute each of the following showing as much work as possible. Round to two decimal places as needed. Check results using your calculator.

a) $\sqrt{49} =$

b) $\sqrt[3]{8} =$

c) $\sqrt{-49} =$

d) $\sqrt[3]{-8} =$

e) $-25^{3/2}$

f) $(-25)^{3/2}$

g) $\sqrt[7]{49}$

h) $\sqrt[4]{12^3}$

Problem 2	**YOU TRY – Compute with Rational/Radical Exponents**

Compute each of the following showing as much work as possible. Round to two decimal places as needed.

a) $\sqrt{36}$

b) $\sqrt[3]{-64}$

c) $16^{3/2}$

d) $(-25)^{1/2}$

e) $\left(\sqrt[3]{27}\right)^4$

f) $\sqrt[9]{81}$

Section 7.2 – Simplifying Rational Exponents, Distance and Midpoint Formulas

Rules of Exponents

1. Product Rule: $x^n x^m = x^{n+m}$
2. Quotient Rule: $\frac{x^n}{x^m} = x^{n-m}$
3. Power Rule: $(x^n)^m = x^{nm}$
4. Negative exponent rule: $x^{-n} = \frac{1}{x^n}$, $\frac{1}{x^{-n}} = x^n$
5. Zero exponent rule, $x \neq 0$: $x^0 = 1$

Assume the variables represent positive real numbers. Simplify, write with positive exponents only and write the radical form of each expression:

Problem 3	WORKED EXAMPLE – Simplifying Rational Exponents

$$x^{1/3}x^{1/2} = x^{\frac{1}{3}+\frac{1}{2}} = x^{\frac{2}{6}+\frac{3}{6}} = x^{\frac{5}{6}} = \sqrt[6]{x^5}$$

Problem 4	YOU TRY – Simplifying Rational Exponents

$x^{1/2}x^{1/4}$

Problem 5	MEDIA EXAMPLE – Simplifying Rational Exponents

a) $\frac{x^{5/2}}{x^{1/2}}$

b) $\frac{t^{\frac{20}{3}}}{t^{\frac{11}{4}}}$

Problem 6 YOU TRY – Simplifying Rational Exponents

$$\frac{x^{1/2}}{x^{1/3}}$$

Problem 7 MEDIA EXAMPLE – Simplifying Rational Exponents

$$\left(x^{\frac{2}{3}}\right)^{2/7}$$

Problem 8 YOU TRY– Simplifying Rational Exponents

$$\left(y^{2/5}\right)^{5}$$

Problem 9 YOU TRY – Simplifying Rational Exponents

$$9^{-1/2}$$

Problem 10 YOU TRY – Simplifying Rational Exponents

$$\left(2x^{\frac{1}{2}}\right)^{4}$$

Distance and Midpoint Formulas

The distance between any two points (x_1, y_1) and (x_2, y_2) on the Cartesian coordinate plane is:

$$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

Midpoint of these two points is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Problem 11	**WORKED EXAMPLE– Distance and Midpoint**

Find the approximate distance and midpoint between the given points (problems 11-15):

(4,5) and (7, 1)

Solution:
$(x_1, y_1) = (4,5)$
$(x_2, y_2) = (7,1)$

$$d = \sqrt{(7-4)^2 + (1-5)^2} = \sqrt{(3)^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4+7}{2}, \frac{5+1}{2}\right) = \left(\frac{11}{2}, \frac{6}{2}\right) = \left(\frac{11}{2}, 3\right)$$

Problem 12	**MEDIA EXAMPLE – Distance and Midpoint**

(-2, 3) and (3, -9)

Problem 13	**YOU TRY – Distance and Midpoint**

(-4, 4) and (6, -6)

Problem 14	**YOU TRY – Distance and Midpoint**

(-2, 5) and (8, 3)

Problem 15	**YOU TRY – Distance and Midpoint**

(-8, -5) and (6, -1)

Section 7.3 – Square Root Functions – Key Characteristics

A basic square root function has the form

$$f(x)=\sqrt{p(x)},$$

where $p(x)$ is a polynomial, and $p(x)\geq 0$.
(Remember that we cannot take the square root of negative numbers in the real number system.)

DOMAIN

To determine the domain of $f(x)$, you want to find the values of x such that $p(x)\geq 0$.

HORIZONTAL INTERCEPT (X-INT)

To determine the Horizontal Intercept (x-int) for the basic square root function $f(x)=\sqrt{p(x)}$, solve the equation $p(x) = 0$. Then write the ordered pair (x,0).

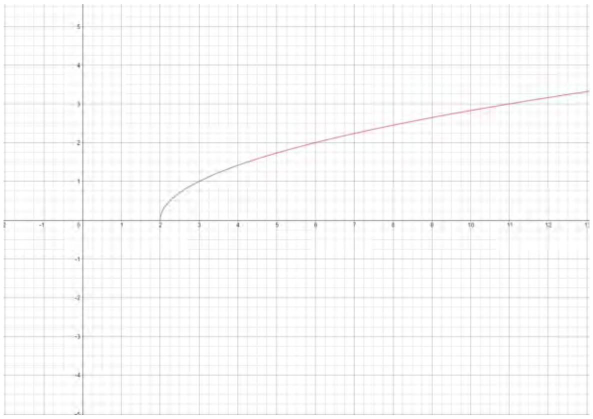
VERTICAL INTERCEPT (Y-INT)

To determine the Vertical Intercept (y-int), evaluate $f(0)$. Then write the ordered pair (0,f(0)).

Problem 16	**WORKED EXAMPLE – Key Characteristics of Square Root Functions**

Graph $f(x)=\sqrt{x-2}$ and determine Vertical Intercept (y-int), Horizontal Intercept (x-int), and domain of f(x).

To graph, input into Y1 the following:
2nd>X^2 then x-2) so that Y1= $\sqrt{(x-2)}$.
Graph on the standard window (Zoom 6):



DOMAIN
Solve $x-2\geq 0$ to get $x\geq 2$.
Therefore the domain is $x\geq 2$.

HORIZONTAL INTERCEPT (X-INT)
Solve $x - 2 = 0$ to get $x = 2$. The Horizontal Intercept (x-int) is (2,0)

VERTICAL INTERCEPT (Y-INT)
Determine f(0) = $\sqrt{0-2}=\sqrt{-2}$
which is not a real number.
There is no Vertical Intercept (y-int).

Problem 17 MEDIA EXAMPLE – Key Characteristics of Square Root Functions

For each of the following, determine the domain, Horizontal Intercept (x-int), and Vertical Intercept (y-int), then sketch an accurate graph of each.

a) $f(x)=\sqrt{4-x}$

Domain of $f(x)$:

Horizontal Intercept (x-int):

Vertical Intercept (y-int):

b) $g(x)=\sqrt{x-4}$

Domain of $g(x)$:

Horizontal Intercept (x-int):

Vertical Intercept (y-int):

Problem 18 YOU TRY – Key Characteristics of Square Root Functions

Given the function $f(x)=\sqrt{12-4x}$, determine the domain, the Horizontal Intercept (x-int), the Vertical Intercept (y-int) (if it exists), and draw an accurate graph of the function. Round to one decimal place
as needed.

Domain of $f(x)$:

Horizontal Intercept (x-int):

Vertical Intercept (y-int):

Section 7.4 – Cube Root Functions – Key Characteristics

A basic cube root function has the form $f(x)=\sqrt[3]{p(x)}$, where $p(x)$ is a polynomial.

To enter f(x) in your calculator you can rewrite the radical as a rational exponent

$$f\left(x\right)=\sqrt[3]{p\left(x\right)}=\left(p\left(x\right)\right)^{\frac{1}{3}}$$

OR

On your TI calculator, press the MATH button then select option 4 for the cube root.

DOMAIN

The domain of $f(x)$ is all real numbers.

HORIZONTAL INTERCEPT (X-INT)

To determine the Horizontal Intercept (x-int) for the basic cube root function $f(x)=\sqrt[3]{p(x)}$,

solve the equation $p(x) = 0$.

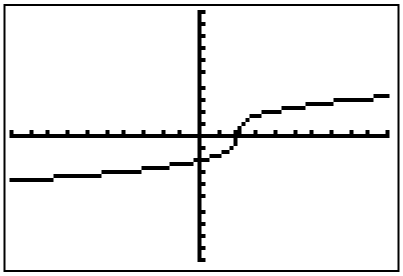
VERTICAL INTERCEPT (Y-INT)

To determine the Vertical Intercept (y-int), evaluate $f(0)$.

Problem 19	**WORKED EXAMPLE – Key Characteristics of Cube Root Functions**

Graph $f(x)=\sqrt[3]{4x-8}$ and determine Vertical Intercept (y-int), Horizontal Intercept (x-int), and domain of $f(x)$.

Graph on the standard window (Zoom 6) to get the graph below:



DOMAIN

The domain is all real numbers.

HORIZONTAL INTERCEPT (X-INT)

Solve $4x – 8 = 0$ to get $x = 2$. The Horizontal Intercept (x-int) is (2,0)

VERTICAL INTERCEPT (Y-INT)

$$f(0)=\sqrt[3]{4(0)-8}=\sqrt[3]{-8}=-2$$

So the Vertical Intercept (y-int) is (0, –2).

Problem 20 MEDIA EXAMPLE – Key Characteristics of Cube Root Functions

For each of the following, determine the domain, Horizontal Intercept (x-int), and Vertical Intercept (y-int), then sketch an accurate graph of each. Round to one decimal place.

a) $f(x)=\sqrt[3]{27-15x}$

Domain of $f(x)$:

Horizontal Intercept (x-int):

Vertical Intercept (y-int):

b) $g(x)=\sqrt[3]{2x-10}$

Domain of $g(x)$:

Horizontal Intercept (x-int):

Vertical Intercept (y-int):

Problem 21 YOU TRY – Key Characteristics of Cube Root Functions

Given the function $f(x)=\sqrt[3]{x+8}$, determine the domain, the Horizontal Intercept (x-int), the Vertical Intercept (y-int) (if it exists), and draw an accurate graph of the function.

Domain of $f(x)$:

Horizontal Intercept (x-int):

Vertical Intercept (y-int):

Section 7.5 – Radical Functions – Key Characteristics

A basic radical function has the form $f(x)=\sqrt[n]{p(x)}$, where $n \geq 2$, n is a whole number and $p(x)$ is a polynomial.

DOMAIN: The DOMAIN of $f(x)=\sqrt[n]{p(x)}$ depends on the value of *n*.

- If *n* is EVEN (like the square root function), then the domain consists of all values of *x* for which $p(x) \geq 0$. Remember that we cannot take an even root of negative numbers in the real number system.
- If *n* is ODD (like the cube root function), then the domain is all real numbers.

HORIZONTAL INTERCEPT (x-int)

To determine the Horizontal Intercept (x-int) for the basic radical function $f(x)=\sqrt[n]{p(x)}$, solve the equation $p(x) = 0$.

VERTICAL INTERCEPT (y-int)

To determine the Vertical Intercept (y-int), evaluate $f(0)$.

Problem 22	**MEDIA EXAMPLE – Key Characteristics of Radical Functions**

For each of the following, determine the domain, Horizontal Intercept (x-int), and Vertical Intercept (y-int), then sketch an accurate graph of each. Write your answers in exact form and give the decimal approximation rounded to the nearest hundredth.

a) $f(x)=\sqrt[4]{x-5}$

Domain:

Horizontal Intercept (x-int):

Vertical Intercept (y-int):

b) $g(x)=\sqrt[7]{11+x}$

Domain:

Horizontal Intercept (x-int):

Vertical Intercept (y-int):

Problem 23	**YOU TRY– Key Characteristics of Radical Functions**

For each of the following, determine the domain, Horizontal Intercept (x-int), and Vertical Intercept (y-int), then sketch an accurate graph of each. Write your answers in exact form and give the decimal approximation rounded to the nearest hundredth.

a) $f(x) = \sqrt[5]{20 - x}$

Domain:

Horizontal Intercept (x-int):

Vertical Intercept (y-int):

b) $g(x) = \sqrt[8]{8 + 4x}$

Domain:

Horizontal Intercept (x-int):

Vertical Intercept (y-int):

Section 7.6 – Solve Radical Equations by Graphing

Solve Radical Equations by Graphing

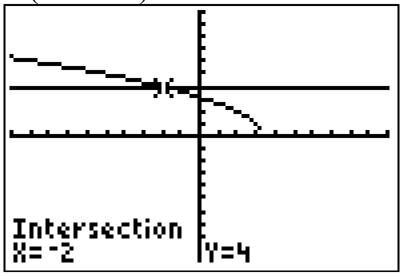
- Let Y1 = one side of the equation
- Let Y2 = other side of the equation
- Determine an appropriate window to see important parts of the graph
- Use the Intersection Method
- *Note: If your graphs do not cross, then there is no intersection and no solution to the equation.*

Problem 24	**WORKED EXAMPLE – Solve Radical Equations by Graphing**

Solve the equation $\sqrt{10-3x}=4$ graphically.

Let Y1 = $\sqrt{10-3x}$
Let Y2 = 4

Graph on the standard window (Zoom:6) then determine the intersection (seen below).



```
Intersection
X=-2          Y=4
```

Your solution is the x-value of the intersection which in this case is $x = -2$.

Problem 25 MEDIA EXAMPLE – Solve Radical Equations by Graphing

Solve the equations graphically. Include a rough but accurate sketch of the graphs and intersection point. Mark and label the intersection. Round answers to two decimal places.

a) $\sqrt[3]{2x-1}=5$

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

Solution: x = ______________

b) $41+5\sqrt{2x-4}=11$

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

Solution: x = ______________

Problem 26 YOU TRY – Solve Radical Equations by Graphing

Solve the equation $\sqrt[5]{8x+133}=4$ graphically. Include a rough but accurate sketch of the graphs and intersection point. Mark and label the intersection. Round answers to two decimal places.

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

Solution: x = ______________

Section 7.7 – Solve Radical Equations Algebraically

To solve radical equations algebraically (also called symbolically):

- Isolate the radical part of the equation on one side and anything else on the other
- Sometimes you will have radicals on both sides. That is ok.
- Raise both sides of the equation to a power that will "undo" the radical (2nd power to get rid of square root, 3rd power to get rid of cube root, etc…)
- Solve.
- *Check your answer! Not all solutions obtained will check properly in your equation.*

Problem 27	**WORKED EXAMPLE – Solve Radical Equations Algebraically**

Solve the equation $\sqrt{10-3x}=4$ algebraically.

First, square both sides to remove the square root.

$$\begin{aligned}\sqrt{10-3x}&=4\\ \left(\sqrt{10-3x}\right)^2&=4^2\\ 10-3x&=16\end{aligned}$$

Next, isolate *x*.

$$\begin{aligned}10-3x&=16\\ -3x&=6\\ x&=-2\end{aligned}$$

VERY IMPORTANT! Check $x = -2$ in the original equation to be sure it works! *Not all solutions obtained using the process above will check properly in your equation.* If an *x* does not check, then it is not a solution**.**

$$\begin{aligned}\sqrt{10-3(-2)}&=4\\ \sqrt{10+6}&=4\\ \sqrt{16}&=4\\ 4&=4\end{aligned}$$

$x = -2$ is the solution to this equation.

Problem 28	**MEDIA EXAMPLE – Solve Radical Equations Algebraically**

Solve the equations algebraically. Write your answers in exact form, then give the decimal approximation rounded to the nearest hundredth.

a) $\sqrt[3]{2x-1}=5$

b) $41+5\sqrt{2x-4}=11$

Problem 29	**YOU TRY – Solve Radical Equations Algebraically**

Solve the equations algebraically. Write your answers in exact form, then give the decimal approximation rounded to the nearest hundredth. Be sure to check your final result!

a) $3\sqrt{4-x}-7=20$

b) $5+\sqrt[4]{3x-1}=7$

c) $2\sqrt{5x}+24=4$

d) $\sqrt[3]{2-5x}-4=6$

Problem 30	**WORKED EXAMPLE – Solve Radical Equations – More Advanced**

Solve the equation algebraically and check graphically: $\sqrt{x+6}=x$. Be sure to check your final result!

Since the radical is isolated, square both sides to remove the square root.

$$\sqrt{x+6}=x$$
$$\left(\sqrt{x+6}\right)^2=x^2$$
$$x+6=x^2$$
$$0=x^2-x-6$$
$$x^2-x-6=0$$

What we now have is a quadratic equation. The easiest and fastest way to work with this problem is through factoring. You can also use the Quadratic Formula or graphing.

$$x^2-x-6=0$$
$$(x+2)(x-3)=0$$
$$x+2=0 \text{ or } x-3=0$$
$$x=-2 \text{ or } x=3$$

CHECK:

When $x=-2$

$$\sqrt{-2+6}=-2?$$
$$\sqrt{4}=-2?$$
$$2\neq-2$$

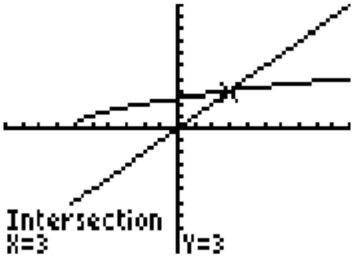
$x=-2$ **does not check so is not a solution**

When $x=3$

$$\sqrt{3+6}=3?$$
$$\sqrt{9}=3?$$
$$3=3$$

$x=3$ checks so is a solution.

Graphical Check: $Y1=\sqrt{x+6}$, $y2=x$ Window: Standard (Zoom:6)



Using the Intersection Method,
we obtain a verified solution of $x = 3$.

Problem 31 MEDIA EXAMPLE – Solve Radical Equations – More Advanced

Solve the equation algebraically ***and*** graphically: $1+\sqrt{7-x}=x$.
Be sure to check your final result!

Problem 32 YOU TRY – Solve Radical Equations – More Advanced

Solve the equation algebraically ***and*** graphically: $\sqrt{x+6}=x+4$.
Be sure to check your final result!

Problem 33	**YOU TRY – Solve Radical Equations – Application Problem**

Voltage through a circuit is determined by the formula $V = \sqrt{PR}$, where P is power, measured in watts, R is the resistance, measured in ohms and V is the voltage, measured in volts. Round answers to two decimal places as needed.

a) Determine the amount of resistance that is required for 8 watts of power to produce 4 volts.

b) Determine the voltage produced if 121 watts is supplied with a resistance of 40 ohms.

c) Determine the amount of power that must be supplied in order to produce 100 volts if the resistance is 10 ohms.

Name:________________________ **Date:__________________**

Lesson 7 Practice Problems

Round to two decimal places unless stated otherwise.

Section 7.1: Roots, Radicals, and Rational Exponents

1. Complete the table below. Each expression should be written in radical notation, written with rational exponents and evaluated using the calculator. The first one is done for you.

	Written in radical notation	Written using rational exponents	Evaluated using the calculator (Rounded to two decimal places)
a)	$\sqrt[3]{9}$	$9^{1/3}$	9^(1/3) ≈ 2.08
b)	$\sqrt[5]{20}$		
c)	$\sqrt[3]{2^4}$		
d)	$-\sqrt[4]{7^2}$		
e)	$\sqrt[3]{(-8)}$		
f)		$3^{1/4}$	
g)		$11^{1/7}$	
h)		$-4^{1/2}$	
i)		$(-2)^{2/3}$	

2. Evaluate the following using your graphing calculator. If there is no *real* solution, write "N". Round answers to three decimal places if necessary.

a) $2\sqrt{9}$

b) $\dfrac{\sqrt[5]{-32}}{5}$

c) $\dfrac{4}{\sqrt[3]{-64}}$

d) $-\sqrt{46}$

e) $\sqrt[4]{(-4)^2}$

f) $\sqrt[4]{-80}$

g) $\sqrt[3]{8^2}$

h) $-\sqrt[3]{8^3}$

Section 7.2: Simplifying Rational Exponents, Distance and Midpoint Formulas

3) Assume the variables represent positive real numbers. Simplify and write with positive exponents only:

a) $x^{\frac{2}{3}}x^{\frac{3}{5}}$

b) $x^{\frac{2}{3}}x^{-2}$

c) $\dfrac{y^{3}}{y^{-\frac{1}{2}}}$

d) $\left(a^{\frac{2}{3}}b^{\frac{1}{4}}\right)^{-2}$

e) $\left(x^{-2}yz^{-\frac{1}{3}}\right)^{-3}$

4) Find the approximate distance and the midpoint between the given points:

a) (-1,2) and (3,4)

b)(-2,1) and (4,4)

c)(3,-5) and (-1,-3)

d) (-1,-2) and (-5,-2)

Section 7.3: Square Root Functions – Key Characteristics

5. Complete the table below.

	Function	*Domain*	*Horizontal Intercept (x-int)*	*Vertical Intercept (y-int)*
a)	$g(x)=\sqrt{x-2}$			
b)	$f(x)=\sqrt{4x-6}$			
c)	$f(x)=2\sqrt{4x+2}$			
d)	$s(t)=\sqrt{3-t}$			
e)	$h(x)=\sqrt{12-6x}$			

6. Use your graphing calculator to complete the table and sketch the graph of each of the functions below. Use an appropriate viewing window.

a) $g(x) = \sqrt{x-2}$

x	$g(x) = \sqrt{x-2}$
2	
3	
6	

---

b) $f(x) = \sqrt{4x-6}$

x	$f(x) = \sqrt{4x-6}$
$\frac{3}{2}$	
$\frac{7}{4}$	
$\frac{5}{2}$	

---

c) $f(x) = 2\sqrt{4x+2}$

x	$f(x) = 2\sqrt{4x+2}$
$-\frac{1}{2}$	
$-\frac{1}{4}$	
0	

d) $s(t) = \sqrt{3 - t}$

t	$s(t) = \sqrt{3 - t}$
0	
2	
3	

e) $h(x) = \sqrt{12 - 6x}$

x	$h(x) = \sqrt{12 - 6x}$
0	
$\frac{11}{6}$	
2	

Section 7.4: Cube Root Functions – Key Characteristics

7. Complete the table below.

	Function	*Domain*	*Horizontal Intercept (x-int)*	*Vertical Intercept (y-int)*
a)	$f(x)=\sqrt[3]{x+8}$			
b)	$f(x)=\sqrt[3]{9-2x}$			

8. Use your graphing calculator to complete the tables and sketch the graphs of the functions below. Use an appropriate viewing window.

a) $f(x)=\sqrt[3]{x+8}$

x	$f(x)=\sqrt[3]{x+8}$
-5	
0	
5	

b) $f(x)=\sqrt[3]{9-2x}$

x	$f(x)=\sqrt[3]{9-2x}$
-5	
0	
5	

Section 7.5: Radical Functions – Key Characteristics

9. For each of the functions below, determine the domain, Horizontal Intercept (x-int), Vertical Intercept (y-int). Then sketch the graph of the function on an appropriate viewing window.

a) $f(x)=\sqrt[5]{8x-32}$

Domain	Horizontal Intercept (x-int)	Vertical Intercept (y-int)	Graph

b) $f(x)=\sqrt{9-2x}$

Domain	Horizontal Intercept (x-int)	Vertical Intercept (y-int)	Graph

c) $f(x)=\sqrt[4]{5x-20}$

Domain	Horizontal Intercept (x-int)	Vertical Intercept (y-int)	Graph

d) $f(x)=\sqrt[3]{4x+8}$

Domain	Horizontal Intercept (x-int)	Vertical Intercept (y-int)	Graph

e) $f(x)=\sqrt[6]{-x}$

Domain	Horizontal Intercept (x-int)	Vertical Intercept (y-int)	Graph

f) $f(x)=\sqrt[3]{1-x}$

Domain	Horizontal Intercept (x-int)	Vertical Intercept (y-int)	Graph

g) $f(x)=\sqrt{4x+11}$

Domain	Horizontal Intercept (x-int)	Vertical Intercept (y-int)	Graph

h) $f(x)=\sqrt[5]{-3x}$

Domain	Horizontal Intercept (x-int)	Vertical Intercept (y-int)	Graph

Section 7.6: Solve Radical Equations by Graphing

10. Solve each of the equations by graphing. Round any decimal results to three places. Sketch the graph on an appropriate viewing window. **Mark and label** the solution(s) on your graph.

a) $6+\sqrt[3]{7-3x}=16$

Xmin=_______ Xmax=________

Ymin=_______ Ymax=________

Solution(s):________________________

b) $\sqrt{3-2x}=14$

Xmin=_______ Xmax=________

Ymin=_______ Ymax=________

Solution(s):________________________

c) $4\sqrt{x-6}=12$

Xmin=_______ Xmax=________

Ymin=_______ Ymax=________

Solution(s):________________________

d) $\sqrt[4]{2x+8}+5=0$

Xmin=_______ Xmax=________

Ymin=_______ Ymax=________

Solution(s):________________________

e) $\sqrt{5-x}-7=2$

Xmin=_______ Xmax=________

Ymin=_______ Ymax=________

Solution(s):_______________________

f) $5-\sqrt[3]{5x}=11$

Xmin=_______ Xmax=________

Ymin=_______ Ymax=________

Solution(s):_______________________

g) $\sqrt{5-x}=x+17$

Xmin=_______ Xmax=________

Ymin=_______ Ymax=________

Solution(s):_______________________

h) $7+\sqrt[3]{15x}=x-8$

Xmin=_______ Xmax=________

Ymin=_______ Ymax=________

Solution(s):_______________________

Section 7.7: Solve Radical Equations Algebraically

11. Solve each of the equations algebraically. Show all of your work. Round any decimal results to three places. Verify that your answer checks with the graphical solutions from problem 8.

a) $6+\sqrt[3]{7-3x}=16$

b) $\sqrt{3-2x}=14$

c) $4\sqrt{x-6}=12$

d) $\sqrt[4]{2x+8}+5=0$

e) $\sqrt{5-x}-7=2$

f) $5-\sqrt[3]{5x}=11$

12. Solve each of the equations algebraically. Show all of your work. Round any decimal results to three places. Check by graphing. Sketch the graph on an appropriate viewing window. **Mark and label** the solution(s) on your graph.

a) $\sqrt{8x-7}=x$

b) $\sqrt{45+4x}=x$

c) $\sqrt{4x}=x-3$

d) $\sqrt{2x+10}+5=x+6$

13. A person's Body Mass Index is calculated with the formula: $BMI = \left(\frac{Weight}{Height^2}\right)703$ where

a) Weight is in pounds and Height is in inches. The equation solved for Height is

$$Height = \sqrt{\frac{703(Weight)}{BMI}}$$

b) Each of the people listed below have a BMI of 30. Use the formula found in part a) to complete the table. Round to the nearest tenth as needed.

Name	Weight	Height
Sara	120	
Leonard		78
Marta	155	
Dillon		65
Mike	250	
Peggy		58

14. Voltage through a circuit is determined by the formula $V = \sqrt{PR}$, where P is power, measured in watts, R is the resistance, measured in ohms and V is the voltage, measured in volts. Round answers to two decimal places as needed.

    a) Determine the amount of resistance that is required for 2 watts of power to produce 4 volts.

    b) Determine the voltage produced if 100 watts is supplied with a resistance of 40 ohms.

    c) Determine the amount of power that must be supplied in order to produce 50 volts if the resistance is 30 ohms.

Name:____________________ **Date:**__________________

Lesson 7 Assessment

1. Evaluate the following using your graphing calculator. If there is no *real* solution, write "N". Round answers to three decimal places if necessary.

$\sqrt[6]{42} =$_______________ $\sqrt[3]{-512} =$_______________ $\sqrt[4]{-625} =$________________

2. Solve the following equations algebraically. Show all steps. Use your graphing calculator to check your answers.

a) $6+\sqrt{7-3x}=16$

b) $\sqrt[4]{2x+8}+5=0$

c) $5-\sqrt[3]{5x}=11$

d) $\sqrt{5-x}=x+1$

3. Multiply $x^{\frac{2}{5}}x^{\frac{1}{3}}$ and write the answer:

A) with a single positive rational exponent:

B) using radical notation

4. Complete the table. Write intercepts as ordered pairs. Use inequality notation for the domain. Round to the nearest hundredth as needed. Write "N" if the answer does not exist

	$f(x)=\sqrt[4]{8x}$	$f(x)=\sqrt{3-x}$	$f(x)=\sqrt[3]{x+8}$
Vertical Intercept (y-int)			
Horizontal Intercept (x-int)			
Domain			
Determine *f*(5)			
Graph			

5. The distance, in feet, travelled by Marina's car when she skidded on an icy patch is modeled by the formula $D = t\sqrt{r}$.

A) Find r, the speed of the car in feet per second, when she skidded for 5 seconds and travelled a distance of 30 ft.

B) Find the distance travelled when the car skidded at a speed of 49 feet per second for 8 seconds.

Lesson 8 – Rational Functions

In this lesson, you will embark on a study of rational functions. These may be unlike any function you have ever seen. Rational functions look different because they are in pieces but understand that the image presented is that of a single function.

In this lesson, you will graph rational functions and solve rational equations both graphically and algebraically. You will finish the lesson with an application of rational functions.

Lesson Topics

Section 8.1: Characteristics of Rational Functions

- Domain
- Vertical Asymptotes
- Horizontal Asymptotes

Section 8.2: Least Common Multiple (LCM), Least Common Denominator (LCD), Simplifying and Adding/Subtracting Rational Functions

Section 8.3: More Operations on Rational Functions

Section 8.4: Solving Rational Equations

- Solve by graphing
- Solve algebraically (LCD Method)
- Determine Horizontal (x-int) and Vertical Intercepts (y-int)
- Working with Input and Output

Section 8.5: Applications of Rational Functions

Mini-Lesson 8

Section 8.1 – Characteristics of Rational Functions

A RATIONAL FUNCTION is a function of the form

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomials and $q(x)$ does not equal zero (remember that division by zero is undefined). Assume, for simplicity, that the rational function is in simplest form when determining the asymptotes. Rational functions have similar shapes depending on the degree of the polynomials $p(x)$ and $q(x)$. However, the shapes are different enough that only the general characteristics are listed here and not a general graph:

DOMAIN

- The solution(s) found when solving $q(x) = 0$ are the values that are NOT part of the domain of $f(x)$.

VERTICAL ASYMPTOTES

- $f(x)$ will have VERTICAL ASYMPTOTES at all input values where $q(x) = 0$. These asymptotes are vertical guiding lines for the graph of $f(x)$
- The graph of $f(x)$ will never cross over these lines.
- To find the Vertical Asymptotes for $f(x)$, set the denominator, $q(x)$, equal to 0 and solve for x. For each solution, x = that value is the equation of your vertical asymptote.

HORIZONTAL ASYMPTOTES

The asymptote is a line the function approaches. If $f(x)$ has a HORIZONTAL ASYMPTOTE $y = a$, then as the input values approach positive and negative infinity, the output values will approach $y=a$.

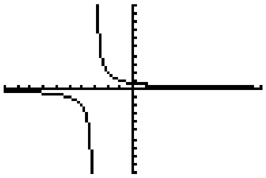
Two methods for finding the Horizontal Asymptote of a Rational Function:

1. Enter $f(x)$ into your graphing calculator, then go to your TABLE, and enter in "large" values for the input. Start with 10, then 100, 1000, and so on. If a horizontal asymptote exists, you will notice the output values "approaching" a number, a. The line $y = a$ is the horizontal asymptote.
2. Make a fraction with only the greatest degree term(the term with the greatest exponent) in $p(x)$ as the numerator and the greatest degree term in $q(x)$ as the denominator. Reduce this fraction completely.
   - **Case I**: If the fraction reduces to a number, then y = that number is the equation of the horizontal asymptote. The degree of $p(x)$ is equal to the degree of $q(x)$
   - **Case II**: If the fraction reduces to $\frac{\text{number}}{x}$, then $y = 0$ is the horizontal asymptote. The degree of $p(x)$ is less than the degree of $q(x)$.
   - **Case III**: $f(x)$ will have a horizontal asymptote only if the degree of $p(x) \leq$ degree of $q(x)$. If $f(x)$ does not have a horizontal asymptote, we say it DNE.

Problem 1	**WORKED EXAMPLE – Key Characteristics of Rational Functions**

Graph $f(x)=\dfrac{2}{x+3}$ and determine the horizontal and vertical asymptotes and the domain.

To graph, let Y1 = $\dfrac{2}{x+3}$. Input Y1=2/(x+3) into your Y= list and note the use of ( ).



To find any VERTICAL ASYMPTOTES, set the denominator equal to 0 and solve for x.
$x + 3 = 0$, therefore $x = –3$. The equation of the vertical asymptote is $x = –3$.

To find the DOMAIN, set the denominator equal to 0 and solve for x. Because $x = –3$ makes the denominator of $f(x)$ equal zero, this value is not part of the domain. All other inputs are allowable. So, the domain for f(x) is $x \neq –3$, "all real numbers except –3".

To find the HORIZONTAL ASYMPTOTE, make a fraction of the highest power term in the numerator (2) and the highest power term in the denominator (x). Reduce. Here is what the fraction looks like.

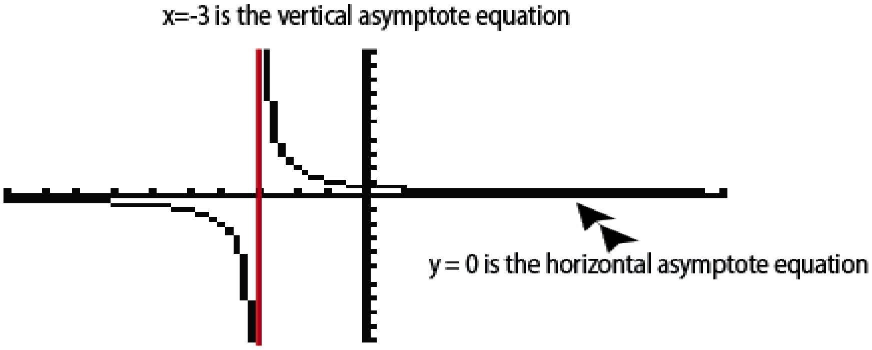
$$\frac{2}{x}$$

What you are trying to find out is, what is the value of this function as x gets really big (positive) and really big (negative)? To answer this question, we need to apply a little abstract thinking.

ABSTRACT THINKING

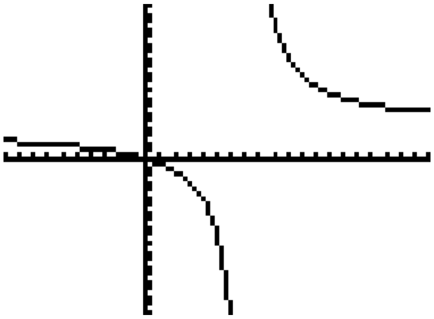
- In your mind, think of the very biggest positive number you can think of. What happens when you divide 2 by that number? Well, the result is going to be very, very small…effectively zero if your number is big enough. So, y = 0 is your horizontal asymptote equation as the same thing works for the biggest negative number you can think of. This is an example of Case II.

Putting all these things together gives the following graph with asymptotes labeled:

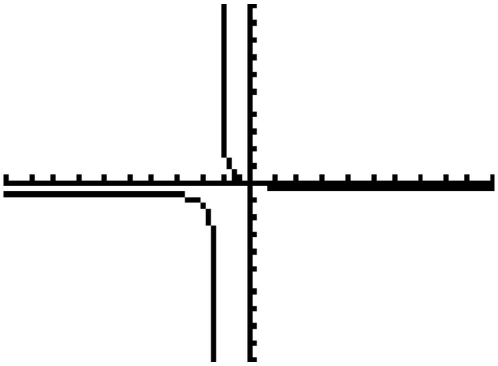


Problem 2	**MEDIA EXAMPLE – Key Characteristics of Rational Functions (Appears as problem 5 online)**

a) Graph $f(x)=\dfrac{4x}{x-7}$ and determine the horizontal and vertical asymptotes and the domain.



b) Graph $f(x)=\dfrac{-3x}{7x+9}$ and determine the horizontal and vertical asymptotes and the domain.



c) **Quick Example:** Find the Horizontal Asymptote for $f(x)=\dfrac{5}{x+1}$.

Problem 3 YOU TRY – Key Characteristics of Rational Functions

a) Graph $f(x)=\frac{4}{x-5}$ and determine the horizontal and vertical asymptotes and the domain. Label the asymptotes on the graph of $f(x)$.

Domain:

Vertical Asymptote:

Horizontal Asymptote:

b) Graph $g(x)=\frac{3x}{2x+1}$ and determine the horizontal and vertical asymptotes and the domain. Label the asymptotes on the graph of $g(x)$.

Domain:

Vertical Asymptote:

Horizontal Asymptote:

c) Graph $h(x)=\frac{2x+1}{4-x}$ and determine the horizontal and vertical asymptotes and the domain. Label the asymptotes on the graph of $h(x)$.

Domain:

Vertical Asymptote:

Horizontal Asymptote:

Section 8.2 – Least Common Multiple (LCM), Least Common Denominator (LCD), and Adding/Subtracting Rational Functions

Definition: A rational function is **simplified** or **reduced** if the greatest common factor of the numerator and denominator is 1.

We can reduce a rational function by factoring the numerator and denominator and then "cancelling" (dividing by) any common factors between the numerator and denominator.

Problem 4	**WORKED EXAMPLE – Simplifying Rational Functions**

$$g(x)=\frac{x^2-9}{5x+15}$$

Step 1: Factor the numerator and denominator, if possible:

$$g(x)=\frac{(x+3)(x-3)}{5(x+3)}$$

Step 2: Divide or Cancel the common factor between the numerator and denominator

$$g(x)=\frac{(x-3)}{5}$$

For the 2nd step, recall: $\frac{(x+3)}{(x+3)}=1, x\neq-3$

Problem 5	**MEDIA EXAMPLE – Simplifying Rational Functions**

a) $g(x)=\frac{2x+6}{2x+10}$

b) $f(x)=\frac{2x^2-6x-20}{2x^2-50}$

Problem 6	**YOU TRY – Simplifying Rational Functions**

$$h(x)=\frac{x+1}{x^2+2x+1}$$

Problem 7	**YOU TRY – Simplifying Rational Functions**

$$q(x)=\frac{2x}{x^2+2x}$$

Definition: The least common multiple (**LCM**) of two or more numbers is the least multiple of the numbers that is the same between the original numbers.

Multiples Method

Problem 8	**WORKED EXAMPLE – Finding the LCM**

Find the LCM of 2 and 3.

Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, …
Multiples of 3: 3, 6, 9, 12, 15, 18, 21,....

Common Multiples: 6, 12, …

Least Common Multiple (LCM) of 2 and 3 is 6.

Prime Factorization Method

Problem 9	**WORKED EXAMPLE – Finding the LCM**

Find the LCM of 10 and 21.

Find the prime factorization: $10 = 2 \cdot 5$ and $21 = 3 \cdot 7$

Multiply the distinct factors with a prime base and the highest power or exponent of each factor between 10 and 21 to find the LCM.

In this case, the distinct (prime) factors are $2, 3, 5, 7$ so the LCM is $2 \cdot 3 \cdot 5 \cdot 7 = 6 \cdot 35 = 210$

Problem 10	**YOU TRY – Finding the LCM**

a)Find the LCM of 6 and 8.

b) Find the LCM of 5 and 15.

Finding the LCM of Algebraic Expressions: Prime Polynomials

Problem 11	**WORKED EXAMPLE – Finding the LCM of Polynomials**

Consider the algebraic expressions x and (x+1). These two algebraic expressions are two distinct prime polynomials, meaning that they are only divisible by 1 and themselves. The prime factor x has an exponent of 1 and the prime factor (x+1) also has an exponent of 1. To find the LCM, as in the prime factorization method for numbers, we multiply together the factors with the greatest power of the distinct prime bases, in other words;

Solution:

$$LCM = x(x+1)$$

Problem 12	**WORKED EXAMPLE – Finding the LCM of Polynomials**

Find the LCM of $x-1$ and $x+1$.

Solution:

For the same reasons as above, we multiply them together:

$$LCM = (x+1)(x-1)$$

Problem 13	**WORKED EXAMPLE – Finding the LCM of Polynomials**

Find the LCM of x^2-9 and $x+3$.

Solution:

Factor the difference of two squares (DOTS):

$$x^2-9=(x+3)(x-3)$$

Since $x+3$ and $x^2-9=(x+3)(x-3)$ share a common prime factor we need only multiply each distinct (different) factor together to obtain the LCM:

$$LCM = (x+3)(x-3)$$

Definition: The Least Common Denominator (LCD) of two or more fractions is the LCM of the denominators.

NOTE: We find the LCD when we are asked to add or subtract two or more fractions with **UNLIKE** denominators.

Problem 14	WORKED EXAMPLE – Adding Rational Numbers - Unlike Denominators

Add: $\frac{5}{6}+\frac{3}{7}$

Solution:

Step 1: Find the LCD: $LCD = 6 \cdot 7 = 42$

Step 2: Re-write each fraction as an equivalent fraction with the LCD.

$$\frac{5}{6}=\frac{5}{6}\cdot\frac{7}{7}=\frac{35}{42} \qquad \frac{3}{7}=\frac{3}{7}\cdot\frac{6}{6}=\frac{18}{42}$$

Step 3: These are the two equivalent fractions, then we add and reduce below:

$$\frac{35}{42}+\frac{18}{42}=\frac{53}{42}$$

Step 4: (Write the final fraction in factored form and simplify, if possible)
Since the GCF (Greatest Common Factor) of the numerator and denominator of the final fraction is 1 this means the fraction is in simplest form and thus we have completed the problem.

Problem 15	**WORKED EXAMPLE – Adding Two Rational Functions – Unlike Monomial Denominators**

Find $f(x)+g(x)$ in simplest form if $f(x)=\frac{5}{6x}$ and $g(x)=\frac{3}{7}$.

Solution:

$$f(x)+g(x)=\frac{5}{6x}+\frac{3}{7}$$

Step 1: Find the LCD:

$$LCD=\left(6x\right)\cdot 7=42x$$

Step 2: Re-write each fraction as an equivalent fraction with the LCD:

$$f(x)=\frac{5}{6x}\cdot\frac{7}{7}=\frac{35}{42x}$$

$$g(x)=\frac{3}{7}\cdot\frac{6x}{6x}=\frac{18x}{42x}$$

Step 3: Now we have common denominators, so we are able to add the functions:

$$f(x)+g(x)=\frac{5}{6x}+\frac{3}{7}=\frac{35}{42x}+\frac{18x}{42x}=\frac{35+18x}{42x}$$

Step 4: (Write the final fraction in factored form and simplify, if possible)
Since the GCF (Greatest Common Factor) of the numerator and denominator of the final fraction is 1 this means the fraction is in simplest form and thus we have completed the problem.

Problem 16	**WORKED EXAMPLE – Subtracting Two Rational Functions – Unlike Monomial Denominators**

Find $f(x)-g(x)$ in simplest form if $f(x)=\frac{2}{3x}$ and $g(x)=\frac{3x}{8}$.

Solution:

$$f(x)-g(x)=\frac{2}{3x}+\frac{3x}{8}$$

WARNING: Reducing (Cancelling) the term 3x at the beginning of the problem is not a valid mathematical simplification. In order to add the functions, find the LCD first.

Step 1: Find the LCD:

$$LCD=\left(3x\right)\cdot 8=24x$$

Step 2: Re-write each fraction as an equivalent fraction with the LCD.

$$f(x)=\frac{2}{3x}\cdot\frac{8}{8}=\frac{16}{24x}$$

$$g(x)=\frac{3x}{8}\cdot\frac{3x}{3x}=\frac{9x^2}{24x}$$

Step 3: Now we have common denominators, so we are able to add the functions:

$$f(x)-g(x)=\frac{2}{3x}-\frac{3x}{8}=\frac{16}{24x}+\frac{-\left(9x^2\right)}{24x}=\frac{16-9x^2}{24x}$$

Step 4: (Write the final fraction in factored form and simplify, if possible)

$$f(x)-g(x)=\frac{16-9x^2}{24x}=\frac{\left(4-3x\right)\left(4+3x\right)}{24x}$$

Since the GCF (Greatest Common Factor) of the numerator and denominator of the final fraction is 1 this means the fraction is in simplest form and thus we have completed the problem.

Problem 17	**MEDIA EXAMPLE – Adding Two Rational Functions – Unlike Denominators**

Find $f(x)+g(x)$ in simplest form if $f(x)=\dfrac{4}{3x}$ and $g(x)=\dfrac{2x}{5}$.

Problem 18	**YOU TRY – Adding Two Rational Functions – Unlike Denominators**

Find $f(x)+g(x)$ in simplest form if $f(x)=\dfrac{3}{5x}$ and $g(x)=\dfrac{5x}{3}$.

Problem 19	**MEDIA EXAMPLE – Subtracting Two Rational Functions – Unlike Denominators**

Find $f(x)-g(x)$ in simplest form if $f(x)=\dfrac{2}{x}$ and $g(x)=\dfrac{x}{x-3}$.

Problem 20	**YOU TRY – Add and Subtract Two Rational Functions – Unlike Denominators**

Find $f(x)+g(x)$ and $f(x)-g(x)$ in simplest form if $f(x)=\frac{3}{x}$ and $g(x)=\frac{x}{x-1}$.

Problem 21	**WORKED EXAMPLE – Adding Two Rational Functions - Trinomial and a Binomial Denominator**

Find $f(x)+g(x)$ in simplest form if $f(x)=\frac{-2}{x^2-6x+5}$ and $g(x)=\frac{5}{x-1}$.

Solution:

$$f(x)+g(x)=\frac{-2}{x^2-6x+5}+\frac{5}{x-1}$$

Step 1: Find the LCD (by factoring the denominators and multiplying the different factors of each denominator): Factor: $x^2-6x+5=(x-5)(x-1)$

$$LCD=(x-5)(x-1)$$

Step 2: Re-write each fraction as an equivalent fraction with the LCD:

$$\frac{-2}{x^2-6x+5}=\frac{-2}{(x-5)(x-1)}$$

$$\frac{5}{x-1}=\frac{5}{(x-1)}\cdot\frac{(x-5)}{(x-5)}=\frac{5(x-5)}{(x-1)(x-5)}$$

Step 3: Now we have common denominators, so we are able to add the functions:

$$f(x)+g(x)=\frac{-2}{(x-5)(x-1)}+\frac{5(x-5)}{(x-1)(x-5)}=\frac{-2+5(x-5)}{(x-1)(x-5)}=\frac{-2+5x-25}{(x-1)(x-5)}=\frac{5x-27}{(x-1)(x-5)}$$

Step 4: (Write the final fraction in factored form and simplify, if possible)

Since the GCF (Greatest Common Factor) of the numerator and denominator of the final fraction is 1 this means the fraction is in simplest form and thus we have completed the problem.

Section 8.3 – More Operations on Rational Functions

New Functions from Old Functions

Problem 22	**WORKED EXAMPLE – Operations on Rational Functions - Common Denominators**

Suppose we are given two rational functions $f(x)=\frac{3}{x}$ and $g(x)=\frac{-7}{x}$ we can add, subtract, multiply, divide, and write the functions in simplest form to create new functions:

Add: $f(x)+g(x)$

$$f(x)+g(x)=\frac{3}{x}+\frac{-7}{x}=\frac{3+(-7)}{x}=\frac{-4}{x}$$

Subtract: $f(x)-g(x)$

$$f(x)-\mathrm{g}(x)=\frac{3}{x}-\frac{-7}{x}=\frac{3-(-7)}{x}=\frac{3+7}{x}=\frac{10}{x}$$

Multiply: $f(x)g(x)$

$$f(x)g(x)=\frac{3}{x}\cdot\frac{-7}{x}=\frac{-21}{x^2}$$

Recall: You don't need to find common denominators with multiplication.

Divide: $\frac{f(x)}{g(x)}$

$$\frac{f(x)}{g(x)}=f(x)\div g(x)=\frac{3}{x}\div\frac{-7}{x}=\frac{3}{x}\cdot\frac{x}{-7}=\frac{3}{-7}$$

Recall: Dividing two fractions is the same as multiplying by the reciprocal of $g(x)$

Problem 23	**YOU TRY – Operations on Rational Functions – Common Denominators**

Suppose that $f(x)=\frac{1}{x}$ and $g(x)=\frac{-4x+1}{x}$, find the following in simplest or reduced form:

Add: $f(x)+g(x)$

Subtract: $f(x)-g(x)$

Multiply: $f(x)g(x)$

Divide: $\dfrac{f(x)}{g(x)}$

In the above examples, there was a common denominator. Suppose that we have **unlike** denominators (for addition/subtraction you may refer back to section 8.2):

Problem 24	**YOU TRY – Operations on Rational Functions**

Suppose that $f(x)=\dfrac{1}{x}$ and $g(x)=\dfrac{-4x}{5}$, find the following in simplest or reduced form:

Add: $f(x)+g(x)$

Subtract: $f(x)-g(x)$

Multiply: $f(x)g(x)$

Divide: $\dfrac{f(x)}{g(x)}$

Problem 25	**YOU TRY – Operations on Rational Functions**

Suppose that $f(x)=\dfrac{x+3}{x+1}$ and $g(x)=\dfrac{5}{6}$, find the following in simplest or reduced form:

Add: $f(x)+g(x)$

Subtract: $f(x)-g(x)$

Multiply: $f(x)g(x)$

Divide: $\dfrac{f(x)}{g(x)}$

Problem 26	**YOU TRY – Operations on Rational Functions**

Suppose that $f(x)=\dfrac{2}{x-2}$ and $g(x)=\dfrac{x}{x+2}$, find the following in simplest or reduced form:

Add: $f(x)+g(x)$

Subtract: $f(x)-g(x)$

Multiply: $f(x)g(x)$

Divide: $\dfrac{f(x)}{g(x)}$

Problem 27	**YOU TRY – Operations on Rational Functions**

Suppose that $f(x)=\dfrac{x^2-6x}{x+4}$ and $g(x)=1$, find the following in simplest or reduced form:

Add: $f(x)+g(x)$

Subtract: $f(x)-g(x)$

Multiply: $f(x)g(x)$

Divide: $\frac{f(x)}{g(x)}$

Problem 28	**YOU TRY – Operations on Rational Functions**

Suppose that $f(x)=\frac{1}{x^2-9}$ and $g(x)=\frac{2}{x+3}$, find the following in simplest or reduced form:

Add: $f(x)+g(x)$

Subtract: $f(x)-g(x)$

Multiply: $f(x)g(x)$

Divide: $\frac{f(x)}{g(x)}$

Section 8.4 – Solving Rational Equations Graphically and Algebraically

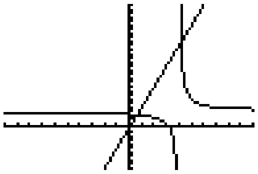
To solve Rational Equations by GRAPHING:

- Let Y1 = one side of the equation
- Let Y2 = other side of the equation
- Determine an appropriate window to see important parts of the graph
- Use the intersection method
- You may have more than one solution
- The *x*-value(s) of the intersection are your solutions

Problem 29	**WORKED EXAMPLE – Solve Rational Equations by Graphing**

Solve $5x = 4 + \frac{3}{x-4}$

Let Y1 = 5*x*
Let Y2 = 4 + 3/(*x* – 4) Note use of ( )
Graph on window x: [–10..10], y:[–10..30]
If you use standard window you do not see the upper intersection.



You will need to perform the intersection process two separate times on your calculator. One time, you should get x = 0.62 (the left intersection) and the second time you should get x = 4.18. Be sure to move your cursor far enough (it has to go all the way across the vertical asymptote) to read the second intersection. Solutions, then, are *x* = 0.62, 4.18

Problem 30	**MEDIA EXAMPLE – Solve Rational Equations by Graphing**

Solve $3 = 1 + \frac{3x}{x-1}$ by graphing.

To solve rational equations ALGEBRAICALLY (also called symbolically):

- Identify the least common denominator (LCD) for all fractions in the equation.
- Take note of the values of x that make the common denominator zero. These x-values cannot be used as solutions to the equation since we cannot divide by 0.
- Clear the fractions by multiplying both sides of the equation by the least common denominator (LCD).
- Solve for x.
- Check your work by plugging the value(s) back into the equation or by graphing.

Problem 31	**WORKED EXAMPLE – Solve Rational Equations Algebraically**

Solve $5x = 4 + \frac{3}{x-4}$ algebraically. Round solutions to two decimal places.

- Common denominator for all sides is x – 4. Multiply both sides of the equation by (x – 4) and solve for x to get the following:

$$(5x)(x-4) = (4 + \frac{3}{x-4})(x-4)$$

$$5x^2 - 20x = 4(x-4) + \frac{3}{x-4}(x-4)$$

$$5x^2 - 20x = 4x - 16 + 3$$

$$5x^2 - 20x = 4x - 13$$

$$5x^2 - 20x - 4x + 13 = 0$$

$$5x^2 - 24x + 13 = 0$$

Notice that we now have a quadratic equation, which can be solved using the methods of last chapter. Because we are asked to solve our original problem algebraically, let's continue that process and not resort to a graphical solution. We will use the Quadratic Formula with a = 5, b = –24, and c = 13 to get:

$$x = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(5)(13)}}{2(5)} = \frac{24 \pm \sqrt{576-260}}{10} = \frac{24 \pm \sqrt{316}}{10}$$

Because we want rounded solutions, I do NOT need to continue reducing my fraction solutions above but can compute the following directly:

$$x = \frac{24 + \sqrt{316}}{10} \approx 4.18, \qquad x = \frac{24 - \sqrt{316}}{10} = .62$$

These solutions match what we found in the graphing example previously.

To check, plug the values back into the original equation (one at a time) or use the graphing method.

Problem 32 MEDIA EXAMPLE – Solving Rational Equations Algebraically

Solve $3=1+\frac{3x}{x-1}$ algebraically. Check your solution.

Problem 33 YOU TRY – Solving Rational Equations Graphically/Algebraically

Round answer(s) to two decimals as needed.

a) Solve $1=\frac{5}{x-2}-3$ graphically. Sketch the graph from your calculator screen, and indicate the viewing window you used.

Xmin=______

Xmax=______

Ymin=______

Ymax=______

Solution: __________________

b) Solve $1=\frac{5}{x-2}-3$ algebraically. Show complete work and check your answer.

Problem 34	**YOU TRY – Solving Rational Equations Algebraically**

Solve $\frac{3}{x}-\frac{4}{x-5}=\frac{2}{x^2-5x}$ algebraically. Check your solution.

Problem 35	**YOU TRY – Solving Rational Equations Algebraically**

Solve $x+\frac{1}{x}=2$ algebraically. Check your solutions.

Problem 36	**YOU TRY – Solving Rational Equations Algebraically**

Solve $\frac{3}{t}+\frac{6}{t^2}=2$ algebraically.

Problem 37	**MEDIA EXAMPLE – Working with Rational Functions**

Consider the function $f(x) = \dfrac{x-1}{x+5}$

a) What is the domain?

b) Give the ***equation*** of the vertical asymptote for $f(x)$.

c) Give the ***equation*** of the horizontal asymptote for $f(x)$.

d) What is the Vertical Intercept (y-int)? Show your work.

e) What is the Horizontal Intercept (x-int)? Show your work.

f) Determine $f(12)$. Show your work.

g) For what value of x is $f(x) = 3$? Show your work.

Problem 38	**YOU TRY – Working with Rational Functions**

Consider the function $g(x)=\dfrac{15x-12}{3x+4}$

a) What is the domain?

b) Give the ***equation*** of the vertical asymptote for $g(x)$.

c) Give the ***equation*** of the horizontal asymptote for $g(x)$.

d) What is the Vertical Intercept (y-int)? Show your work.

e) What is the Horizontal Intercept (x-int)? Show your work.

f) Determine $g(5)$. Show your work.

g) For what value of x is $g(x) = –8$? Show your work.

Section 8.5 – Applications of Rational Functions

Problem 39 MEDIA EXAMPLE – Applications of Rational Functions

You and your family are heading out to San Diego on a road trip. From Phoenix, the trip is 354.5 miles according to Google. Answer the following questions based upon this situation.

a) Use the relationship, Distance = Rate times Time or $d = rT$, to write a rational function $T(r)$ that has the rate of travel, r (in mph), as its input and the time of travel, T (in hours) as its output. The distance will be constant at 354.5 miles.

b) Provide a rough but accurate sketch of the graph in the space below. Label your horizontal and vertical axes. You only need to graph the first quadrant information. Indicate the graphing window you chose.

c) If you average 60 mph, how long will the trip take?

Xmin=______ Xmax=_______ Ymin=_______ Ymax=_______

d) If the trip took 10 hours, what was your average rate of travel?

e) What does the graph indicate will happen as your rate increases?

f) What does the graph indicate will happen as your rate gets close to zero?

Problem 40 YOU TRY – Applications of Rational Functions

You and your friends are heading out to Las Vegas on a road trip. From Scottsdale, the trip is 308.6 miles according to Google. Answer the following questions based upon this situation.

a) Use the relationship, Distance = Rate times Time or $d = rT$, to write a rational function $T(r)$ that has the average rate of travel, r (in mph), as its input and the time of travel (in hours) as its output.

b) Provide a rough but accurate sketch of the graph in the space below. Label your horizontal and vertical axes. You only need to graph the first quadrant information. Indicate the graphing window you chose.

Xmin=______

Xmax=______

Ymin=______

Ymax=______

c) According to Google, the trip should take 5 hours. Determine your average rate of travel if the trip takes 5 hours.

d) Determine the vertical asymptote for $T(r)$, and write a sentence explaining its significance in this situation.

e) Determine the horizontal asymptote for $T(r)$, and write a sentence explaining its significance in this situation.

Problem 41 WORKED EXAMPLE – Operations on Rational Functions

A student can complete their homework 4 times as fast as another student. If the students work together they can complete the same homework in 45 minutes.

How many minutes does it take each student to complete their homework?

Solution (Algebraic and Graphical approach):

Let t represent the amount of time (in minutes) it takes student A, the faster student, to complete their homework.

	Amount of time to complete the homework (in minutes)	Fraction of the homework completed in 1 minute	Total fraction of the homework completed in 1 minute	Solve
Student A	t	$\frac{1}{t}$	$\frac{1}{45}$	$\frac{1}{t}+\frac{1}{4t}=\frac{1}{45}$
Student B	$4t$	$\frac{1}{4t}$		

$$\frac{1}{t}+\frac{1}{4t}=\frac{1}{45}$$

$$180t\left(\frac{1}{t}+\frac{1}{4t}\right)=\left(\frac{1}{45}\right)180t \text{ (Multiply each side by the LCD)}$$

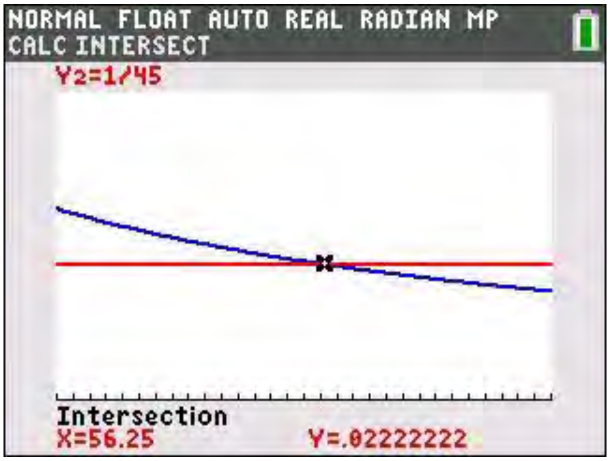
$$\frac{180t}{t}+\frac{180t}{4t}=\frac{180t}{45} \text{ (Distribute and Multiply then Simplify)}$$

$$180+45=4t \text{ (Simplify)}$$

$$225=4t \text{ (Solve for t)}$$

$$\frac{225}{4}=t$$

$$56.25\,\text{min}=\text{t}$$



Working alone, the student A completes the homework in 56.25 minutes.
Working alone, the student B completes the homework in 4(56.25)=225 minutes.

Problem 42 YOU TRY – Operations on Rational Functions

Sue & Jim, working together, can rake the yard in 2 hours. Working alone, Sue can rake the yard in 3.5 hours.

a) How long would it take Jim to rake the yard if he worked alone (include units)? Explain your answer in a complete sentence.

b) State the practical domain.

c) Sketch the situation described in a) below using your calculator and label the point of intersection.

Problem 43	**YOU TRY – Operations on Rational Functions**

In an electrical circuit with resistors placed in parallel, the reciprocal of the total resistance is equal to the sum of the reciprocals of each resistance:

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$$

a) If $R_1 = 20$ Ohms and $R_2 = 10$ Ohms then what is the total resistance, R , in ohms (include units)?

b)If the total resistance is $R = 22$ Ohms and $R_2 = 30$ Ohms then what is resistance, R_1 , in ohms (include units)?

Problem 44	**YOU TRY – Operations on Rational Functions**

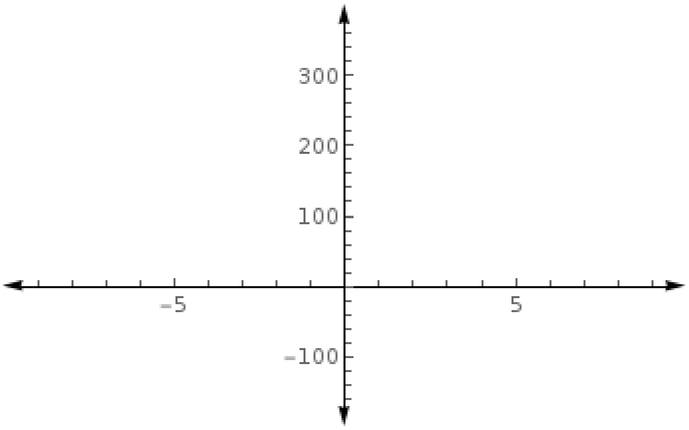
Suppose a battery factory has revenue modeled by the function $R(x)=\dfrac{50x^2}{x+1}$ (in thousands of dollars) where x represents the number of batteries sold. The cost of each battery is modeled by the function $C(x)=\dfrac{50}{x+1}$ in thousands of dollars.

a) What is the cost to produce 3 batteries (include units)?

b) Write the equation that represents the profit in simplest form, $P(x)=R(x)-C(x)$

c) After how many batteries are sold is the profit 200 thousand dollars (include units)?

d) Sketch the situation described in c) below using your calculator and label the point of intersection.



Lesson 8 Practice Problems

Round to two decimal places unless stated otherwise.

Section 8.1: Characteristics of Rational Functions

1. Complete the table below.

	Function	Domain	Vertical Asymptote	Horizontal Asymptote
a)	$f(x)=\frac{4x}{6-2x}$			
b)	$f(x)=\frac{8x+2}{3x-9}$			
c)	$s(t)=\frac{6t+4}{t}$			
d)	$p(t)=\frac{t}{12t-6}$			
e)	$f(x)=\frac{2x-1}{4x-2}$			
f)	$g(x)=\frac{3x-4}{2x+3}$			
g)	$f(x)=\frac{8}{3x}$			

Section 8.2: Least Common Multiple (LCM), Least Common Denominator (LCD), Simplifying, and Adding/Subtracting Rational Functions

2. Simplify:

a) $f\left(x\right)=\dfrac{x-2}{x^2-4x+4}$

b) $g\left(x\right)=\dfrac{x^2-25}{x+5}$

c) $h\left(x\right)=\dfrac{5x^2-80}{5x-20}$

3.Find the LCD of the following functions and find $f(x)+\mathrm{g}(x)$ in simplest form :

a) $g(x)=\dfrac{5}{x+1}$ and $f(x)=\dfrac{5}{x}$

b) $g(x)=\dfrac{-1}{x^2-4}$ and $f(x)=\dfrac{x}{x-2}$

c) $f(x)=\frac{5x}{12}$ and $g(x)=\frac{x+1}{30}$

d) $g(x)=\frac{5x}{7}$ and $f(x)=\frac{5}{x^2-5x+4}$

e) $g(x)=\frac{3x}{5}$ and $f(x)=\frac{5}{3x}$

Section 8.3: More Operations on Rational Functions

4.Suppose that $f(x)=\frac{3}{x}$ and $g(x)=\frac{x}{3}$ perform the indicated operations and write the function in simplest form:

Add: $f(x)+g(x)$

Subtract: $f(x)-g(x)$

Multiply: $f(x)g(x)$

Divide: $\frac{f(x)}{g(x)}$

5.Suppose that $f(x) = \frac{x}{x+4}$ and $g(x) = \frac{x}{x-4}$ perform the indicated operations and write the function in simplest form:

Add: $f(x)+g(x)$

Subtract: $f(x)-g(x)$

Multiply: $f(x)g(x)$

Divide: $\frac{f(x)}{g(x)}$

6.Suppose that $g(x)=\frac{2}{x-2}$ and $f(x)=\frac{x}{x+1}$ perform the indicated operations and write the function in simplest form:

Add: $f(x)+g(x)$

Subtract: $f(x)-g(x)$

Multiply: $f(x)g(x)$

Divide: $\frac{f(x)}{g(x)}$

Section 8.4: Solving Rational Equations

7. Solve each of the following equations by graphing. Round answer(s) to two decimals as needed.

a) $4=\frac{3x}{x-1}$

Solution:__________________

b) $3=2+\frac{5}{x-2}$

Solution:__________________

c) $x+2=\frac{2}{x^2-4}$

Solution:__________________

d) $3=x+\frac{1}{x}$

Solution:__________________

e) $\frac{5}{x+4}=\frac{12}{13}$

Solution:__________________

f) $\frac{2}{x}=1$

Solution:__________________

8.Solve each of the following rational equations algebraically (also called symbolically). Check your work by plugging the value(s) back into the equation or by graphing.

a) $4 = \frac{3}{x-6}$

b) $\frac{4}{x+4} = \frac{6}{x-2}$

c) $\frac{4-2x}{3} = \frac{3x+2}{4}$

d) $6 = 2 + \frac{3}{x-5}$

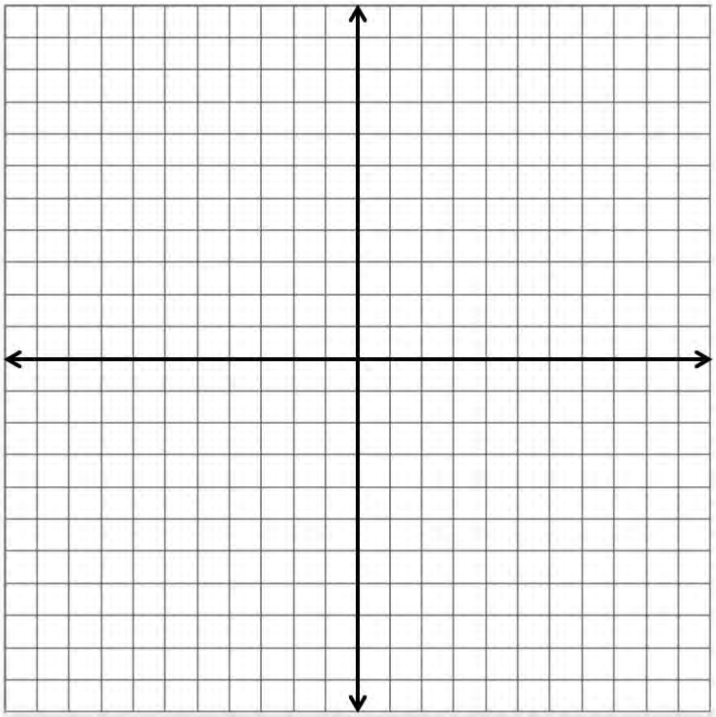
e) $x + 4 = \frac{-4}{x}$

f) $\frac{-1}{x-3} = \frac{x+3}{5}$

9.Graph the rational functions. Make sure to include the domain and any horizontal asymptotes, vertical asymptotes, Horizontal Intercept (x-int) and Vertical Intercept (y-int) (if they exist).

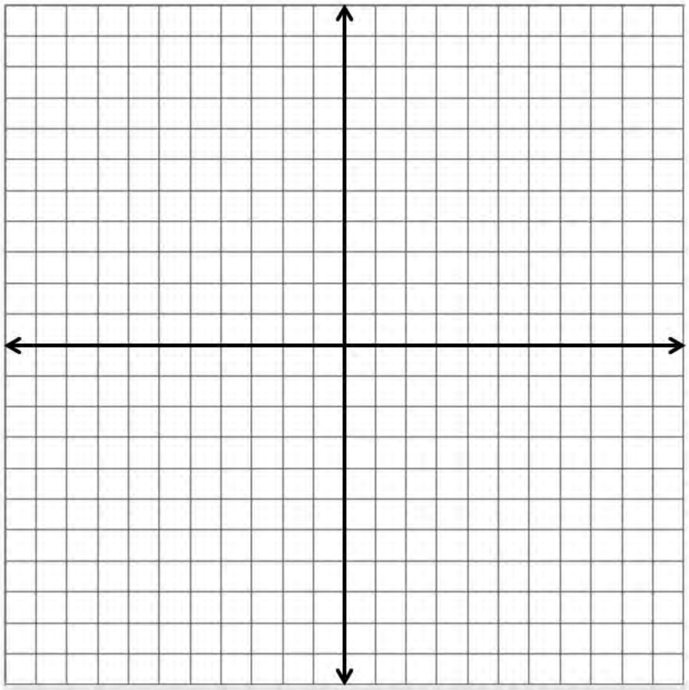
a) $f(x) = \frac{3}{x-2}$

Domain	Horizontal Asymptote	Vertical Asymptote	Horizontal Intercept(s)	Vertical Intercept



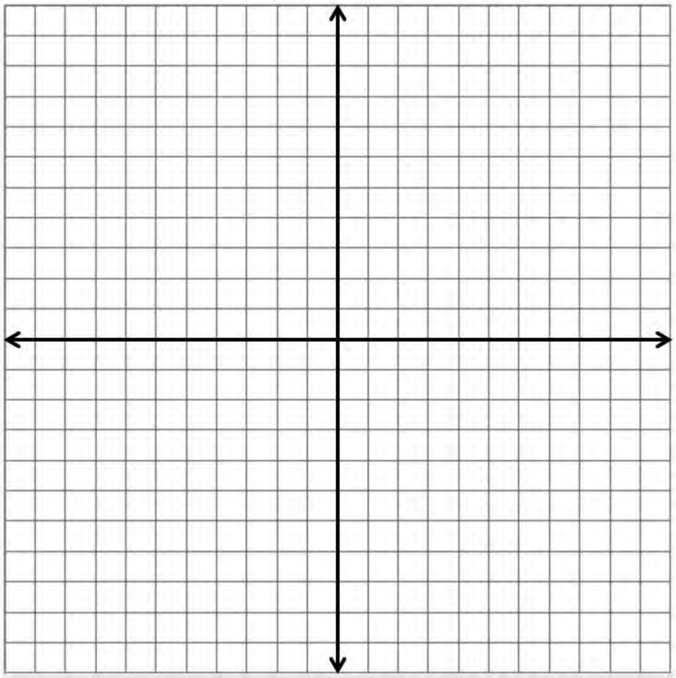
b) $f(x)=\frac{3x+4}{x+3}$

Domain	Horizontal Asymptote	Vertical Asymptote	Horizontal Intercept(s)	Vertical Intercept



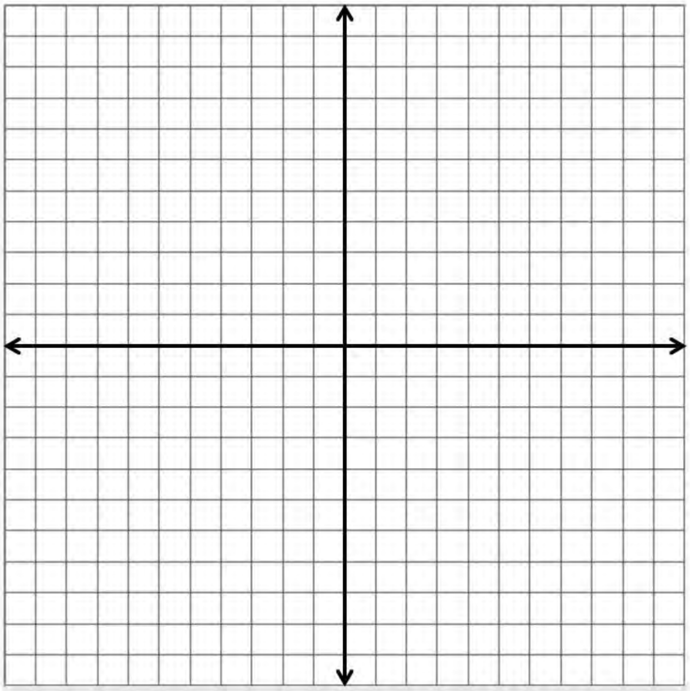
c) $f(x) = \dfrac{x+7}{4-2x}$

Domain	Horizontal Asymptote	Vertical Asymptote	Horizontal Intercept(s)	Vertical Intercept



d) $f(x)=\dfrac{2x-1}{x^2-2x}$

Domain	Horizontal Asymptote	Vertical Asymptotes	Horizontal Intercept(s)	Vertical Intercept



10. Determine the following.

a)Let $f(x)=\frac{3}{x-2}$.

Find $f(-1)$ Find x so that $f(x)=6$

b)Let $h(x)=\frac{3x+4}{x+3}$

Find $h(-2)$ Find x so that $h(x)=\frac{16}{7}$

c)Let $f(x)=\frac{x+7}{4-2x}$.

Find $f(1)$ Find x so that $f(x)=-3$

d)Let $g(x)=\frac{2x-1}{x(x-2)}$ Find x so that $g(x)=\frac{7}{8}$

Find $g(3)$

Section 8.5: Applications of Rational Functions

11.Mr. Sculley decides to make and sell Left Handed Smoke Shifters as a side business. The fixed cost to run his business is $250 per month and the cost to produce each Smoke Shifter averages $8. The Smoke Shifters will sell for $19.95. The function below gives the *average* cost (in dollars) per hat when *x* hats are produced.

$$A(x) = \frac{8x + 250}{x}$$

a)Determine $A(1)$, and write a sentence explaining the meaning of your answer.

b)Complete the table below.

x	1	2	3	4	5
$A(x)$					

c)Determine $A(10)$, and write a sentence explaining the meaning of your answer.

d)How many Smoke Shifters must be produced in order to reduce the average cost to $15 each?

e) Give the *equation* of the horizontal asymptote of $A(x)$, and write a sentence explaining its significance in this situation.

12.You and your friends are heading out to San Diego on a road trip. From Scottsdale, the trip is 373 miles. Answer the following questions based upon this situation.

a)Use the relationship, Distance = Rate times Time or $d = rT$, to write a rational function $T(r)$ that has the rate of travel, r (in mph), as its input and the time of travel (in hours) as its output.

b)Provide a rough but accurate sketch of the graph in the space below. Label your horizontal and vertical axes. You only need to graph the first quadrant information. Indicate the graphing window you chose.

Xmin=______

Xmax=______

Ymin=______

Ymax=______

c)According to Google, the trip should take 5 hours and 45 minutes (5.75 hours). Determine your average rate of travel if the trip takes only 5 hours.

d)Determine the horizontal asymptote for $T(r)$, and write a sentence explaining its significance in this situation.

13.Harkins Theaters offers $1.50 soft drink refills every time you bring your 2013 Harkins Loyalty Cup to the theater. You can purchase the Loyalty Cup (filled) for $6.50. The function $C(x)=\frac{1.5x+6.5}{x}$ gives the average cost (in dollars) per refill with the Loyalty Cup, where x is the number of soft drink refills purchased.

a)Determine $C(1)$, and write a sentence explaining the meaning of your answer.

b)Complete the table below.

x	1	2	3	4	5
$C(x)$					

c)How many refills must you purchase in order to reduce the average cost to $2 per refill?

d) Give the *equation* of the horizontal asymptote of $C(x)$, and write a sentence explaining its significance in this situation.

14. The Crazy Candy Factory has revenue modeled by the function $R(x)=\dfrac{x^2}{x+10}$ (in dollars) where x represents the number of boxes of candy sold and the cost of each box of candy is modeled by the function $C(x)=\dfrac{100}{x+10}$ (in dollars).

a) What is the cost to produce 10 boxes of candy (include units)?

b) Write the equation that represents the profit as a single fraction in simplest form, $P(x)=R(x)-C(x)$

c) After how many boxes of candy are sold is the profit $75 (include units)?

d) Sketch the situation found in c) below using your calculator and label the point of intersection.

Xmin=______

Xmax=______

Ymin=______

Ymax=______

15.A new engine is 3 times more efficient than an old engine. The engines deplete the same tank of gas in 4 hours. (Hint: refer to the yard work problem)

a) How many hours does it take each engine to deplete the tank of gas?

b) Sketch the situation found in a) below using your calculator and label the point of intersection

Xmin=______

Xmax=______

Ymin=______

Ymax=______

Lesson 8 Assessment

1. Consider the function $g(x) = \dfrac{2x-4}{x+5}$

a) What is the domain? ____

b) Give the ***equation*** of the vertical asymptote for $g(x)$. ________________________

c) Give the ***equation*** of the horizontal asymptote for $g(x)$. ______________________

d) What is the Vertical Intercept (y-int)? _____________________

What is the Horizontal Intercept (x-int)? ___________________________

e) For what value of x is $g(x) = 3$? Show your work.

f) Determine $g(42)$. Show your work. Round your answer to three decimal places.

g) Graph and label the asymptotes and intercepts.

Xmin=______

Xmax=______

Ymin=______

Ymax=______

2. You and your family are driving to Santa Fe, NM on a road trip. From Phoenix, the trip is 526 miles according to Google. Answer the following questions based upon this situation. Round to the nearest tenth as needed.

   a) Use the relationship, Distance = Rate times Time or $d = rT$, to write a rational function $T(r)$ that has the average rate of travel, r (in mph), as its input and the time of travel (in hours) as its output. The distance will be constant at 526 miles.

   b) If you average 55 mph, how long will the trip take?

   c) If the trip took 12 hours, what was your average rate of travel?

   d) Determine the practical domain and practical range (include units).

   e) Give the *equation* of the vertical asymptote of $T(r)$, and write a sentence explaining its significance in this situation.

   f) Give the *equation* of the horizontal asymptote of $T(r)$, and write a sentence explaining its significance in this situation.

3. Simplify: $f(x)=\frac{x^2+20x+100}{x^2-100}$

4. Perform the indicated operations given that $f(x)=\frac{2}{x}$ and $g(x)=\frac{x+1}{x-4}$:

Add: $f(x)+g(x)$

Subtract: $f(x)-g(x)$

Multiply: $f(x)g(x)$

Divide: $\frac{f(x)}{g(x)}$

Lesson 9 – Introduction to Exponential Functions

Exponential Functions play a major role in our lives. Many of the challenges we face involve exponential change and can be modeled by an Exponential Function. Financial considerations are the most obvious, such as the growth of our retirement savings, how much interest we are paying on our home loan or the effects of inflation.

In this lesson, we begin our investigation of Exponential Functions by comparing them to Linear Functions, examining how they are constructed and how they behave. We then learn methods for solving exponential functions given the input and given the output.

Lesson Topics:

Section 9.1: Linear Functions Vs. Exponential Functions

- Characteristics of linear functions
- Comparing linear and exponential growth
- Using the common ratio to identify exponential data
- Horizontal Intercepts (x-int)

Section 9.2: Characteristics of Exponential Functions

Section 9.3: Solving Exponential Equations by Graphing

- Using the Intersect Method to solve exponential equations on the graphing calculator
- Guidelines for setting an appropriate viewing window

Section 9.4: Applications of Exponential Functions

Mini-Lesson 9

Section 9.1 – Introduction to Exponential Functions

Problem 1	**YOU TRY – Linear and Exponential Functions**

Investment A starts with $1 and will double each month that passes. After each month, write the amount that you would have up to 12 months. Investment B starts with $500 and will accrue an additional $100 each month. Which investment should you choose over a 12 month time period? Complete the table below:

Time (t in months)	Amount of Money (in dollars), Investment A	Amount of Money (in dollars), Investment B
0	1	500
1	2	600
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

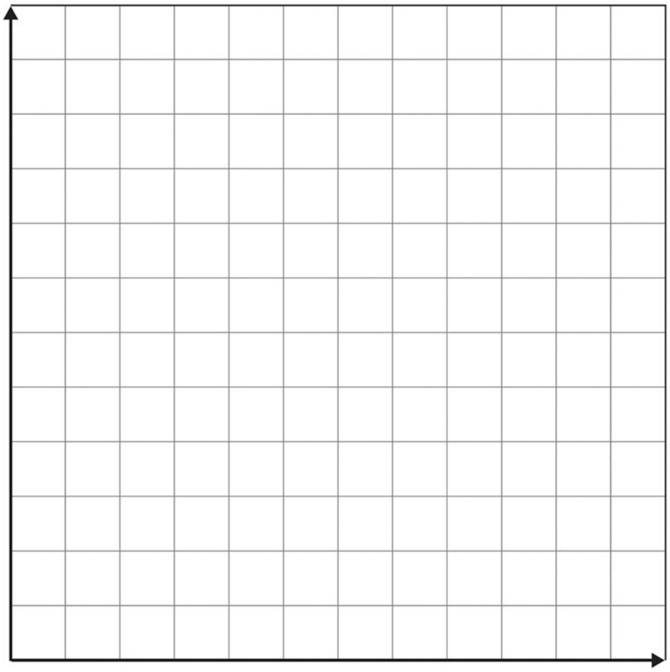
a) Write the equation of the function that represents investment A, A(t), as a function of time (t in months). Is this function linear or non-linear? Explain your answer in a complete sentence.

b) Write the equation of the function that represents investment B, B(t), as a function of time (t in months). Is this function linear or non-linear? Explain your answer in a complete sentence.

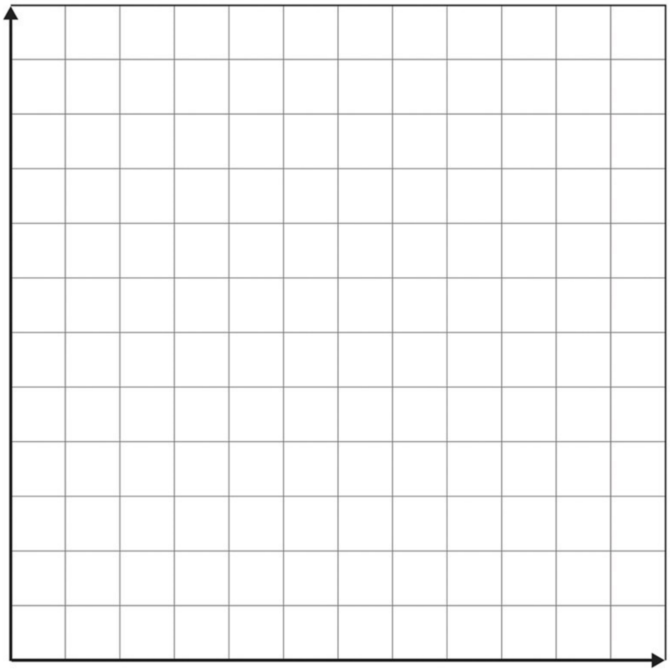
c) What is the amount of money after 24 months for each investment? Use the equations found in a) and b) to evaluate the functions.

d) Plot A(t) and B(t) for time up to and including 5 months. Label your axes appropriately.

Investment A:



Investment B:



e) Based on the graphs above, what are the practical domains and ranges of the functions A(t) and B(t)?

Domain of A(t)

Domain of B(t)

Range of A(t)

Range of B(t)

Definition: An exponential function is a function in the form $f(x) = ab^x$ where $a \neq 0$ and and x is a real number. The number b is called the base (or common ratio), x is called the exponent, and a is called the initial value.

- If $b > 1$, f(x) is a growth (increasing) function.
- If $0 < b < 1$, f(x) is a decay (decreasing) function.

Problem 2 | YOU TRY – Exponential Functions

Investigate the function $f(x) = 3^x$:

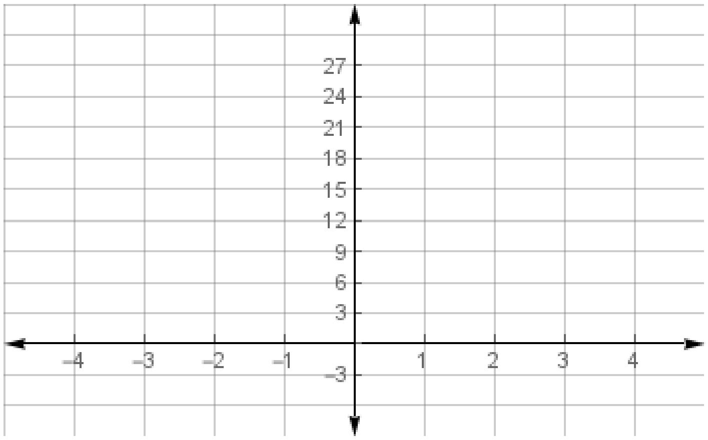
a) Circle One: Exponential Growth or Exponential Decay since ______________________.

b) State the initial value:__________________.

c) Complete the x-y table below (You may use the table on the TI-84, round to one decimal place as needed):

x	$y = f(x) = 3^x$	(x, y)
-3	$3^{-3} = \frac{1}{3^3} = \frac{1}{27}$	$\left(-3, \frac{1}{27}\right)$
-2		
-1		
0		
1		
2		
3		

Plot the points found above on the graph below and sketch the graph:



Problem 3	**YOU TRY – Exponential Functions**

Investigate the function $g(x) = \left(\frac{1}{2}\right)^x$:

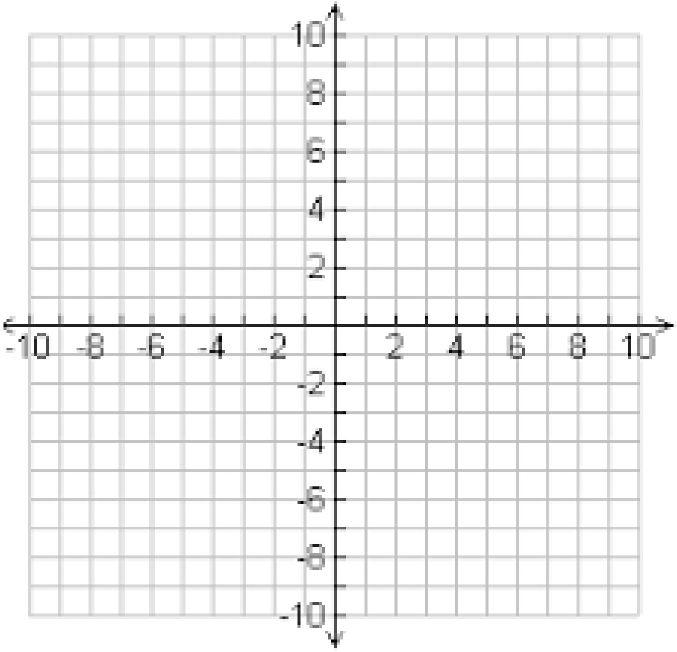
a)Circle One: Exponential Growth or Exponential Decay since ______________________.

b)State the initial value:_________________.

c)Complete the x-y table below (You may use the table on the TI-84):

x	$y = g(x) = \left(\frac{1}{2}\right)^x$	(x, y)
-2	$\left(\frac{1}{2}\right)^{-2} = 4$	$(-2, 4)$
-1		
0		
1		
2		

Plot the points found above on the graph below and sketch the graph:



Problem 4 YOU TRY – Characteristics of Linear Functions

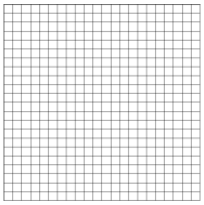
Given a function, $f(x) = mx + b$, respond to each of the following. Refer back to previous lessons as needed.

a) The variable x represents the __________________ quantity.

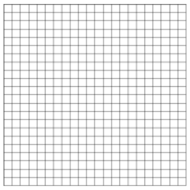
b) $f(x)$ represents the _________________ quantity.

c) The graph of f is a _________________________________ with slope __________ and Vertical Intercept (y-int) _________________ .

d) On the graphing grid below, draw an INCREASING linear function. In this case, what can you say about the slope of the line? m ______ 0 (Your choices here are > or <)



e) On the graphing grid below, draw a DECREASING linear function. In this case, what can you say about the slope of the line? m ______ 0 (Your choices here are > or <)



f) The defining characteristic of a LINEAR FUNCTION is that the AVERAGE RATE OF CHANGE (also called the SLOPE) is _______________________.

g) The domain of a LINEAR FUNCTION is _____________________________________

This next example is long but will illustrate the key difference between EXPONENTIAL FUNCTIONS and LINEAR FUNCTIONS.

Problem 5	**WORKED EXAMPLE – DOLLARS & SENSE**

On December 31st around 10 pm, you are sitting quietly in your house watching Dick Clark's New Year's Eve special when there is a knock at the door. Wondering who could possibly be visiting at this hour you head to the front door to find out who it is. Seeing a man dressed in a three-piece suit and tie and holding a briefcase, you cautiously open the door.

The man introduces himself as a lawyer representing the estate of your recently deceased great uncle. Turns out your uncle left you some money in his will, but you have to make a decision. The man in the suit explains that you have three options for how to receive your allotment.

Option A: $1000 would be deposited on Dec 31st in a bank account bearing your name and each day an additional $1000 would be deposited (until January 31st).

Option B: One penny would be deposited on Dec 31st in a bank account bearing your name. Each day, the amount would be doubled (until January 31st).

Option C: Take $30,000 on the spot and be done with it.

Given that you had been to a party earlier that night and your head was a little fuzzy, you wanted some time to think about it. The man agreed to give you until 11:50 pm. Which option would give you the most money after the 31 days???

A table of values for option A and B are provided on the following page. Before you look at the values, though, which option would you select according to your intuition?

Without "doing the math" first, I would instinctively choose the following option (circle your choice):

Option A	Option B	Option C

Option A: $1000 to start + $1000 per day		Option B: $.01 to start then double each day	
Note that t = 0 on Dec. 31st			
Table of input/output values		Table of input/output values	
t = time in # of days since Dec 31	$A(t)$=$ in account after t days	t = time in # of days since Dec 31	$B(t)$ = $ in account after t days
0	1000	0	.01
1	2000	1	.02
2	3000	2	.04
3	4000	3	.08
4	5000	4	.16
5	6000	5	.32
6	7000	6	.64
7	8000	7	1.28
8	9000	8	2.56
9	10,000	9	5.12
10	11,000	10	10.24
11	12,000	11	20.48
12	13,000	12	40.96
13	14,000	13	81.92
14	15,000	14	163.84
15	16,000	15	327.68
16	17,000	16	655.36
17	18,000	17	1,310.72
18	19,000	18	2,621.44
19	20,000	19	5,242.88
20	21,000	20	10,485.76
21	22,000	21	20,971.52
22	23,000	22	41,943.04
23	24,000	23	83,886.08
24	25,000	24	167,772.16
25	26,000	25	335,544.32
26	27,000	26	671,088.64
27	28,000	27	1,342,177.28
28	29,000	28	2,684,354.56
29	30,000	29	5,368,709.12
30	31,000	30	10,737,418.24
31	32,000	31	21,474,836.48

WOWWWWW!!!!!!!

What IS that number for Option B? I hope you made that choice… it's 21 million, 4 hundred 74 thousand, 8 hundred 36 dollars and 48 cents. Let's see if we can understand what is going on with these different options.

Problem 6 MEDIA EXAMPLE – Compare Linear and Exponential Growth

For the example discussed in Problem 5, respond to the following:

a) Symbolic representation (model) for each situation:

$A(t) =$ Type of function ___________________

$B(t) =$ Type of function ___________________

$C(t) =$ Type of function ___________________

b) Provide a rough but accurate sketch of the graphs for each function on the same grid below:

c) What are the practical domain and range for each function?

	Practical Domain	Practical Range
$A(t)$:		
$B(t)$:		
$C(t)$:		

d) Based on the graphs, which option would give you the most money after 31 days?

e) Let's see if we can understand WHY option B grows so much faster. Let's focus just on options A and B. Take a look at the data tables given for each function. Just the later parts of the initial table are provided.

$A(t) = 1000t + 1000$

t = time in # of days since Dec 31	$A(t)$=$ in account after t days
20	21,000
21	22,000
22	23,000
23	24,000
24	25,000
25	26,000
26	27,000
27	28,000
28	29,000
29	30,000
30	31,000
31	32,000

$B(t) = .01(2)^t$

t =time in # of days since Dec 31	$B(t)$ = $ in account after t days
20	10,485.76
21	20,971.52
22	41,943.04
23	83,886.08
24	167,772.16
25	335,544.32
26	671,088.64
27	1,342,177.28
28	2,684,354.56
29	5,368,709.12
30	10,737,418.24
31	21,474,836.48

As t increases from day 20 to 21, describe how the outputs change for each function:

$A(t)$:

$B(t)$:

As t increases from day 23 to 24, describe how the outputs change for each function:

$A(t)$:

$B(t)$:

So, in general, we can say as the inputs increase from one day to the next, then the outputs for each function:

$A(t)$:

$B(t)$:

In other words, $A(t)$ grows ______________________ and $B(t)$ grows ______________________.

We have just identified the primary difference between **LINEAR FUNCTIONS** and **EXPONENTIAL FUNCTIONS.**

Exponential Functions vs. Linear Functions

The outputs for **Linear Functions** change by **ADDITION** and the outputs for **Exponential Functions** change by **MULTIPLICATION**.

Problem 7 WORKED EXAMPLE – Are the Data Exponential?

To determine if an exponential function is the best model for a given data set, calculate the ratio $\frac{y_2}{y_1}$ for each of the consecutive points. If this ratio is approximately the same for the entire set, then an exponential function models the data best. For example:

x	1	2	3	4	5
y	1.75	7	28	112	448

For this set of data, $\frac{y_2}{y_1} = \frac{7}{1.75} = \frac{28}{7} = \frac{112}{28} = \frac{448}{112} = 4$

Since $\frac{y_2}{y_1} = 4$ for all consecutive pairs, the data are exponential with a growth factor (base) of 4.

Problem 8	**YOU TRY – Use Common Ratio (Base) to Identify Exponential Data**

a) Given the following table, explain why the data can be best modeled by an exponential function. Use the idea of common ratio in your response.

x	0	1	2	3	4	5	6
$f(x)$	15	12	9.6	7.68	6.14	4.92	3.93

b) Determine an exponential model $f(x) = ab^x$ that fits these data. Start by identifying the values of a and b and then write your final result using proper notation.

c) Determine $f(10)$. Round to the nearest hundredth.

d) Determine $f(50)$. Write your answer as a decimal *and* in scientific notation. Round to the nearest hundredth.

Section 9.2 – Characteristics of Exponential Functions

Important Characteristics of the EXPONENTIAL FUNCTION $f(x) = ab^x$

- x represents the INPUT quantity
- $f(x)$ represents the OUTPUT quantity
- The graph of $f(x)$ is in the shape of the letter "J" with Vertical Intercept (y-int) $(0, a)$ and base b (note that b is the same as the COMMON RATIO from previous examples)
- Another way to identify the Vertical Intercept (y-int) is to evaluate $f(0)$ to obtain the ordered pair (0, f(0))

Problem 9	**WORKED EXAMPLE – Examples of Exponential Functions**

a) $f(x) = 2(3)^x$

Initial Value, $a = 2$, Vertical Intercept (y-int) = (0, 2)
Base, $b = 3$.
$f(x)$ is an exponential GROWTH function since $b > 1$.

b) $g(x) =1523(1.05)^x$

Initial Value, $a = 1523$, Vertical Intercept (y-int) = (0, 1523)
Base, $b = 1.05$.
$g(x)$ is an exponential GROWTH function since $b > 1$.

c) $h(x) = 256(0.85)^x$

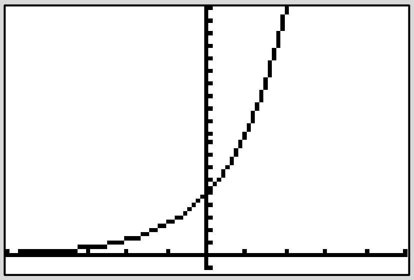
Initial Value, $a = 256$, Vertical Intercept (y-int) = (0, 256)
Base, $b = 0.85$.
$h(x)$ is an exponential DECAY function since $b < 1$.

d) $k(x) = 32(0.956)^x$

Initial Value, $a = 32$, Vertical Intercept (y-int) = (0, 32)
Base, $b = 0.956$.
$k(x)$ is an exponential DECAY function since $b < 1$.

Graph of a generic Exponential Growth Function

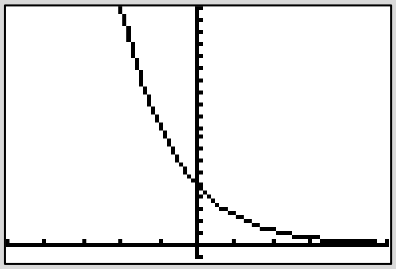
$f(x) = ab^x,\ b > 1,\ a>0$



- Domain: All Real Numbers or $(-\infty, \infty)$
- Range: $f(x) > 0$ or $(0, \infty)$
- Horizontal Intercept (x-int): None
- Vertical Intercept (y-int): $(0, a)$
- Horizontal Asymptote: $y = 0$
- Left to right behavior of the function: INCREASING

Graph of a generic Exponential Decay Function

$f(x) = ab^x,\ 0 < b < 1,\ a>0$



- Domain: All Real Numbers or $(-\infty, \infty)$
- Range: $f(x) > 0$ or $(0, \infty)$
- Horizontal Intercept (x-int): None
- Vertical Intercept (y-int): $(0, a)$
- Horizontal Asymptote: $y = 0$
- Left to right behavior of the function: DECREASING

Problem 10	**MEDIA EXAMPLE – Characteristics of Exponential Functions**

Consider the function $f(x) = 12(1.45)^x$

Initial Value (a): ____________

Base (b):_____________

Domain:_____________________________________

Range:______________________________________

Horizontal Intercept (x-int): ______________________

Vertical Intercept (y-int):_______________________

Horizontal Asymptote: ________________________

Increasing or Decreasing? ______________________

Problem 11	**YOU TRY – Characteristics of Exponential Functions**

Complete the table. Start by graphing each function using the indicated viewing window. Sketch what you see on your calculator screen.

	$f(x) = 335(1.25)^x$	$g(x) = 120(0.75)^x$
Graph Use Viewing Window: Xmin = -10 Xmax = 10 Ymin = 0 Ymax = 1000		
Initial Value (*a*)?		
Base (*b*)?		
Domain? (Use Inequality Notation)		
Range? (Use Inequality Notation)		
Horizontal Intercept (x-int)?		
Vertical Intercept (y-int)?		
Horizontal Asymptote? (Write the *equation*)		
Increasing or Decreasing?		

Section 9.3 – Solving Exponential Equations Graphically and Algebraically

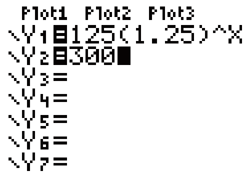
Problem 12	**WORKED EXAMPLE – Solving Exponential Equations by Graphing**

Solve the equation $125(1.25)^x = 300$. Round your answer to two decimal places.

To do this, we will use a process called the INTERSECTION METHOD on our graphing calculators.

To solve $125(1.25)^x = 300$

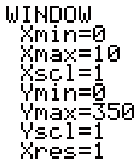
- Press Y= then enter Y1 = $125(1.25)^x$ and Y2 = 300
  Note: You could also let Y1 = 300 and Y2=$125(1.25)^x$



```
Plot1 Plot2 Plot3
\Y1■125(1.25)^X
\Y2■300■
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

- Press WINDOW then enter the values at right.

Try to determine why these values were selected. You must see the intersection in your window. Other entries will work. If you graph and do not see both graphs AND where they intersect, you must pick new WINDOW values until you do.

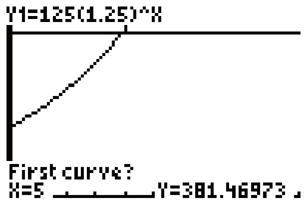


```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=350
 Yscl=1
 Xres=1
```

- Press 2nd>CALC
- Scroll to 5: INTERSECT and press ENTER

Notice the question, "First Curve?" The calculator is asking if Y1 = $125(1.25)^x$ is the first curve in the intersection.

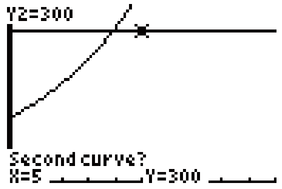
- Press Enter to indicate "Yes"



```
Y1=125(1.25)^X
First curve?
X=5          Y=381.46973
```

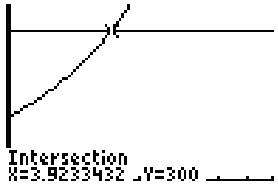
Notice the question, "Second Curve?" The calculator is asking if Y2 = 300 is the second curve in the intersection.

- Press Enter to indicate "Yes"



```
Y2=300
Second curve?
X=5          Y=300
```

- Press Enter at the "Guess" question and obtain the screen at right. Your intersection values are given at screen bottom and the intersection is marked with a cursor. Round as indicated in your problem directions or as dictated by the situation.



```
Intersection
X=3.9233432  Y=300
```

Our answer is $x = 3.92$. Note that this information corresponds to the ordered pair (3.92, 300).

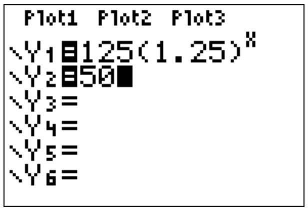
Problem 13 WORKED EXAMPLE – Solving Exponential Equations by Graphing

Given $f(x) = 125(1.25)^x$ find x when $f(x) = 50$. Round your response to two decimal places.

To do this, we need to SOLVE the equation $125(1.25)^x = 50$ using the INTERSECTION METHOD.

To solve $125(1.25)^x = 50$

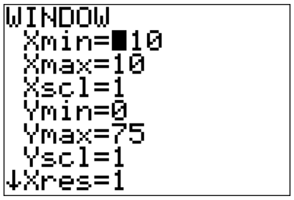
- Press Y= then enter Y1 = $125(1.25)^x$ and Y2 = 50
  Note: You could also let Y1 = 50 and Y2=$125(1.25)^x$



```
Plot1 Plot2 Plot3
\Y1=125(1.25)^X
\Y2=50
\Y3=
\Y4=
\Y5=
\Y6=
```

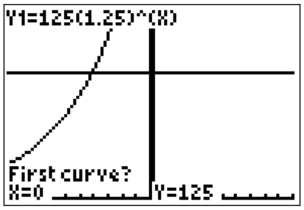
- Press WINDOW then enter the values at right.

Try to determine why these values were selected. You must see the intersection in your window. Other entries will work. If you graph and do not see both graphs AND where they intersect, you must pick new WINDOW values until you do.



```
WINDOW
 Xmin=■10
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=75
 Yscl=1
↓Xres=1
```

- Press 2nd>CALC
- Scroll to 5: INTERSECT and press ENTER



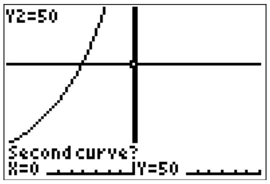
```
Y1=125(1.25)^(X)
First curve?
X=0          Y=125
```

Notice the question, "First Curve?" The calculator is asking if Y1 = $125(1.25)^x$ is the first curve in the intersection.

- Press Enter to indicate "Yes"

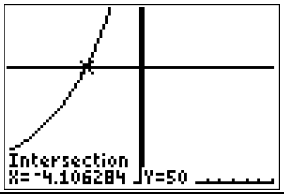
Notice the question, "Second Curve?" The calculator is asking if Y2 = 50 is the second curve in the intersection.

- Press Enter to indicate "Yes"



```
Y2=50
Second curve?
X=0          Y=50
```

- Press Enter at the "Guess" question and obtain the screen at right. Your intersection values are given at screen bottom and the intersection is marked with a cursor. Round as indicated in your problem directions or as dictated by the situation.



```
Intersection
X=-4.106284  Y=50
```

For this problem, we were asked to find x when $f(x) = 50$. Round to two decimal places. Our response is that, "When $f(x) = 50$, $x = -4.11$". Note that this information corresponds to the ordered pair (-4.11, 50) on the graph of $f(x) = 125(1.25)^x$

GUIDELINES FOR SELECTING WINDOW VALUES FOR INTERSECTIONS

While the steps for using the INTERSECTION method are straightforward, choosing values for your window are not always easy. Here are some guidelines for choosing the edges of your window:

- First and foremost, the intersection of the equations MUST appear clearly in the window you select. Try to avoid intersections that appear just on the window's edges, as these are hard to see and your calculator will often not process them correctly.
- Second, you want to be sure that other important parts of the graphs appear (i.e. where the graph or graphs cross the y-axis or the x-axis).
- When choosing values for x, start with the standard XMin = -10 and Xmax = 10 UNLESS the problem is a real-world problem. In that case, start with Xmin=0 as negative values for a world problem are usually not important. If the values for Xmax need to be increased, choose 25, then 50, then 100 until the intersection of graphs is visible.
- When choosing values for y, start with Ymin = 0 unless negative values of Y are needed for some reason. For Ymax, all graphs need to appear on the screen. So, if solving something like $234(1.23)^x = 1000$, then choose Ymax to be bigger than 1000 (say, 1500).

If the intersection does not appear in the window, then try to change only one window setting at a time so you can clearly identify the effect of that change (i.e. make Xmax bigger OR change Ymax but not both at once). Try to think about the functions you are working with and what they look like and use a systematic approach to making changes.

Problem 14	**MEDIA EXAMPLE – Solving Exponential Equations by Graphing**

Solve the equation $400 = 95(0.89)^x$. Round your answer to two decimal places.

Problem 15 | YOU TRY – Window Values and Intersections

In each situation below, you will need to graph to find the solution to the equation using the INTERSECTION method described in this lesson. Fill in the missing information for each situation. Include a rough but accurate sketch of the graphs and intersection point. Mark and label the intersection. Round answers to two decimal places.

a) Solve $54(1.05)^x = 250$ Solution: $x =$ ______________

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

b) Solve $2340(0.82)^x = 1250$ Solution: $x =$ ______________

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

c) Solve $45 = 250(1.045)^x$ Solution: $x =$ ______________

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

Like-Bases Property of Exponential Equations

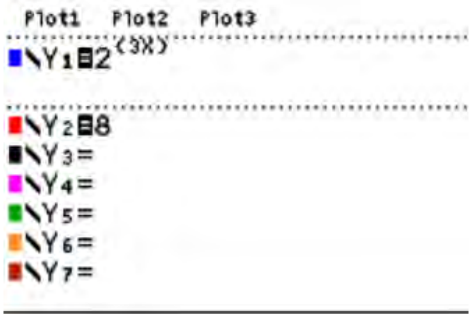
$b^x = b^y$ *if and only if* $x = y$

Problem 16 | **WORKED EXAMPLE – Exponential Functions**

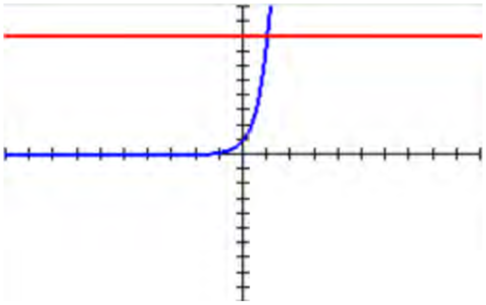
Solve using your calculator and algebraically: $2^{3x} = 8$

Algebraically:

Using the TI-84: (Intersect Method)



```
Plot1  Plot2  Plot3
\Y1=2^(3X)
\Y2=8
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



Solve:

$2^{3x} = 8$

Step 1: Write each side with the same base:

$2^{3x} = 2^3$ (Since $2^3 = 8$)

Step 2: Set the exponents equal to each other:

$3x = 3$

Step 3: Solve the equation:

$3x = 3$

$\frac{3x}{3} = \frac{3}{3}$

$x = 1$

Step 4: CHECK!!!!!!!!!

$2^{3x} = 8$

$2^{3(1)} = 8$

$2^3 = 8$

$8 = 8$

Problem 17 | **MEDIA EXAMPLE – Exponential Functions**

Solve using your calculator and algebraically: $4^{2x-1} = 64$

Algebraically:

$4^{2x-1} = 64$

Using the TI-84 (Label intersection):

Problem 18 YOU TRY– Exponential Functions

Solve using your calculator and algebraically: $4^{x-4} = 16$

Algebraically:

Using the TI-84 (Label Intersection):

Problem 19 YOU TRY – Exponential Functions

Solve using your calculator and algebraically: $3^{2x-9} = 27$

Algebraically:

Using the TI-84 (Label Intersection):

Section 9.4 – Applications of Exponential Functions

Writing Exponential Equations/Functions

Given a set of data that can be modeled using an exponential equation, use the steps below to determine the particulars of the equation:

1. Identify the initial value. This is the *a* part of the exponential equation $y = ab^x$. To find *a*, look for the starting value of the data set (the output that goes with input 0).
2. Identify the common ratio, *b*, value. To do this, make a fraction of two consecutive outputs (as long as the inputs are separated by exactly 1). We write this as the fraction $\frac{y_2}{y_1}$ to indicate that we put the second *y* on top and the first on the bottom. Simplify this fraction and round as the problem indicates to obtain the value of *b*.
3. Plug in the values of *a* and *b* into $y = ab^x$ to write the exponential equation.
4. Replace *y* with appropriate notation as needed to identify a requested exponential FUNCTION.

Problem 20 **MEDIA EXAMPLE – Writing Exponential Equations/Functions**

The population of a small city is shown in the following table.

Year	Population
2000	12,545
2001	15,269
2002	18,584

Assume that the growth is exponential. Let $t = 0$ represent the year 2000. Let *a* be the initial population in 2000. Let *b* equal the ratio in population between the years 2000 and 2001.

a) Write the equation of the exponential model for this situation. Round any decimals to two places. Be sure your final result uses proper function notation.

b) Using this model, forecast the population in 2008 (to the nearest person).

c) Also using this model, determine the nearest whole year in which the population will reach 50,000 using the calculator.

Problem 21	**YOU TRY – Writing Exponential Equations/Functions**

You have just purchased a new car. The table below shows the value, V, of the car after *n* years.

n = number of years	V = Value of Car
0	24,800
1	21,328
2	18,342

a) Assume that the depreciation is exponential. Write the equation of the exponential model for this situation. Round any decimals to two places. Be sure your final result uses proper function notation.

b) You finance the car for 60 months. What will the value of the car be when the loan is paid off? Show all steps. Write your answer in a complete sentence.

Problem 22	**YOU TRY – Writing Exponential Equations/Functions**

In 2010, the population of Gilbert, AZ was about 208,000. By 2011, the population had grown to about 215,000.

a) Assuming that the growth is exponential, construct an exponential model that expresses the population, *P*, of Gilbert, AZ x years since 2010. Your answer must be written in function notation. Round to three decimals as needed.

b) Use this model to predict the population of Gilbert, AZ in 2014. Write your answer in a complete sentence.

c) According to this model, *in what year* will the population of Gilbert, AZ reach 300,000 using the calculator? (Round your answer DOWN to the nearest whole year.)

Problem 23	**YOU TRY – Applications of Exponential Functions**

One 8-oz cup of coffee contains about 100 mg of caffeine. The function $A(x) = 100(0.88)^x$ gives the amount of caffeine (in mg) remaining in the body x hours after drinking a cup of coffee. Answer in complete sentences.

a) Identify the Vertical Intercept (y-int) of this function. Write it as an ordered pair and interpret its meaning in a complete sentence.

b) How much caffeine remains in the body 8 hours after drinking a cup of coffee? Round your answer to two decimal places as needed.

c) How long will it take the body to metabolize half of the caffeine from one cup of coffee? (i.e. How long until only 50mg of caffeine remain in the body?) Show all of your work, use the calculator to solve, and write your answer in a complete sentence. Round your answer to two decimal places as needed.

d) According to this model, how long will it take for *all* of the caffeine to leave the body?

Problem 24 | YOU TRY– Exponential Functions

A certain bank compounds interest on the balances annually in all savings accounts. The function they use to compute the balance is $f(t)=P(1+r)^t$ where P is the principal, t is the time in years, and r is the annual percentage rate (APR). If a savings account has a balance of $5,000 at a 3% annual interest rate use the function to find (show work and round to the nearest hundredth):

a) The account balance after 4 years:

b) The amount of interest after 4 years:

c) Using the TI-84 calculator, after what time (t in years), will the account have $11,400? Round to the hundredths place and sketch the situation below.

<u>Window:</u>

X min:_________________

X max:_________________

Y min:_________________

Y max:_________________

Problem 25 | YOU TRY– Exponential Functions

A certain bank compounds interest on the balances monthly in all savings accounts. The function they use to compute the balance is $g(t) = P\left(1+\frac{r}{n}\right)^{nt}$ where P is the principal and t is the time in years, r is the annual percentage rate (APR), and n is the number of times the interest is compounded in a year. If a savings account has a balance of $5,000 and the interest rate is 2%, use the function to find (show work and round to the nearest hundredth):

a) The account balance after 4 years:

b) The amount of interest after 4 years:

c) Using the TI-84 calculator, after what time (t in years), will the account have $6,400? Round to the hundredths place and sketch the situation below.

Window:

X min:_________________

X max:_________________

Y min:_________________

Y max:_________________

Lesson 9 Practice Problems

Round to two decimal places unless stated otherwise.

Section 9.1: Linear and Exponential Functions

1. Investigate the function $g(x) = \left(\frac{1}{3}\right)^x$:

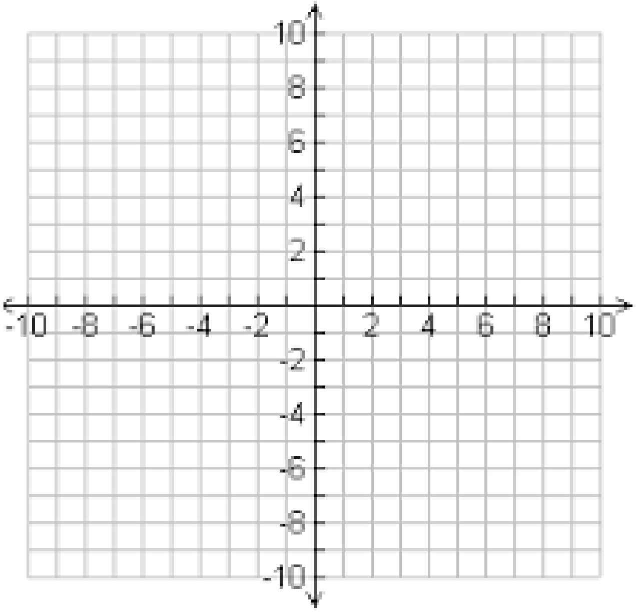
a)Circle One: Exponential Growth or Exponential Decay since ______________________.

b)State the initial value:__________________.

c)Complete the x-y table below (You may use the table on the TI-84):

x	$y = g(x) = \left(\frac{1}{3}\right)^x$	(x, y)
-2		
-1		
0		
1		
2		

Plot the points found above on the graph below and sketch the graph:



2. Complete the table below.

Function	Linear or Exponential?	Linear: find the slope Exponential: find the base	Linear: Increasing or Decreasing? Exponential: Growth or Decay?	Identify the Vertical Intercept (y-int) as an Ordered Pair.
$f(x)=2x+4$	Linear	Slope = 2	Increasing	(0,4)
$f(x)=3(2)^x$				
$g(x)=-1.5x-2$				
$p(t)=100(1.2)^t$				
$f(c)=1.8c+32$				
$g(x)=1000(0.75)^x$				

3. For the following three linear functions, identify the Vertical Intercept (y-int), calculate the slope and then write the equation for the function in $f(x) = mx + b$ form.

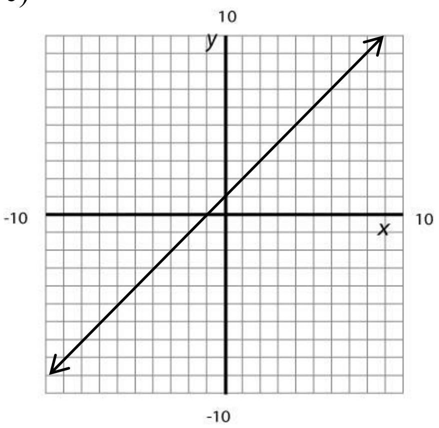
a)

x	$f(x)$
-1	3
0	2
1	1

b)

$f(x) = \{(-2,2), (0,3), (2,4)\}$

c)



$f(x)$

4. For the following three exponential functions, identify the initial value (a), calculate the base (b), and then write the equation for the function in $f(x) = ab^x$ form.

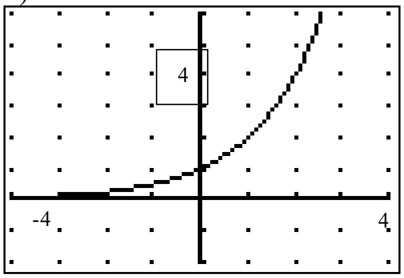
a)

x	$f(x)$
0	4
1	8
2	16

b)

$f(x) = \{(0, 2), (1, 4.2), (2, 8.82)\}$

c)



$f(x)$

5. Determine if each data set is linear or exponential, and write the formula for each. Show complete work.

a)

x	−2	−1	0	1	2	3	4
$f(x)$	.04	.2	1	5	25	125	625

b)

x	−2	−1	0	1	2	3	4
$f(x)$	−1.375	−.5	.375	1.25	2.125	3	3.875

c)

x	−2	−1	0	1	2	3	4
$f(x)$	−3	−5.5	−8	−10.5	−13	−15.5	−18

d)

x	−2	−1	0	1	2	3	4
$f(x)$	98.224	99.108	100	100.9	101.81	102.72	103.65

e)

x	0	1	2	3	4	5	6
$f(x)$	2	4	8	16	32	64	128

Section 9.2: Characteristics of Exponential Functions

6. Complete the table below for each exponential function.

	$f(x) = 3.4(1.13)^x$	$g(x) = 42(0.62)^x$	$h(x) = 1000(1.03)^x$
Initial Value (a)			
Base (b)			
Domain			
Range			
Horizontal Intercept (x-int)			
Vertical Intercept (y-int)			
Horizontal Asymptote			
Increasing or Decreasing			
Growth or Decay			

Section 9.3: Solving Exponential Equations Graphically and Algebraically

7. Given $f(x)=50(1.25)^x$, determine each of the following and show complete work.

a) $f(5) =$

b) $f(50) =$

c) Find x when $f(x) = 75$

d) Find x when $f(x) = -25$

8. Given $f(x)=100(0.90)^x$, determine each of the following and show complete work.

a) $f(3) =$

b) $f(30)=$

c) Find x when $f(x) = 25$

d) Find x when $f(x) = 50$

9. Given $f(x)=25(3)^x$, determine each of the following and show complete work.

a) $f(1) =$

b) $f(3)=$

c) Find x when $f(x) = 100$

d) Find x when $f(x) = 5000$

10. In each situation below, you will need to graph to find the solution to the equation using the INTERSECTION method described in this lesson. Fill in the missing information for each situation. Include a rough but accurate sketch of the graphs and intersection point. Mark and label the intersection. Round answers to two decimal places.

a) Solve $25(1.25)^x = 400$ Solution: x = ______________

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

b) Solve $300(0.85)^x = 80$ Solution: x = ______________

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

c) Solve $300(0.85)^x = 1700$ Solution: x = ______________

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

d) Solve $17.5(2.05)^x = 1$ Solution: $x =$ _______________

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

e) Solve $2(1.01)^x = 12$ Solution: $x =$ _______________

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

f) Solve $532(0.991)^x = 100$ Solution: $x =$ _______________

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

11.For the following equations,

i) Solve algebraically

ii) Graphically using a TI-84 calculator

a) $3^{2x-1}=9$

b) $16=2^{3y-1}$

c) $6^{4y+1}=216$

d) $49^{2x}=343$

e) $8=2^{-2x+1}$

f) $\left(\frac{1}{5}\right)^{-4x}=25$

Section 9.4: Applications of Exponential Functions

12. The rabbit population in several counties is shown in the following table. Assume this growth is exponential. Let $t = 0$ represent the year 2006. Let a represent the initial population in 2006. Let b represent the ratio in population between the years 2006 and 2007.

	Rabbit Population		
Year	Coconino	Yavapai	Harestew
2006	15000	8000	25000
2007	18000	12800	18750
2008	21600	20480	14063
2009	25920	32768	10547

a) Write the equation of the exponential mathematical model for each situation. Round any decimals to two places. Be sure your final result uses proper function notation. Use $C(t)$ for Coconino, $Y(t)$ for Yavapai and $H(t)$ for Harestew.

b) Use the models from part a) to forecast the rabbit population in 2012 for each county. Round to the nearest rabbit. Use proper function notation to represent each result.

c) Use the models from part a) to find the following. Show complete work.

i. The Rabbit Population in Coconino County reaches 60,000.

ii. The Rabbit Population in Yavapai Country reaches 340,000.

iii. The Rabbit Population in Harestew falls below 5000.

d) Which of the scenarios from part c) happened first? Explain your reasoning.

13. Assume you can invest \$1000 at 5% Simple Interest or 4% Compound Interest (Annual). The equation for Simple Interest is modeled by: $A = P + Prt$. Compound Interest is modeled by $A = P(1+r)^t$. The corresponding equations for these two types of interest are given below.

$S(t) = 1000 + 50t$ $C(t) = 1000(1.04)^t$

a) Complete the table for each function.

t	1	5	10	15	20
$S(t)$					
$C(t)$					

b) What is the Vertical Intercept (y-int) for each function and what does it represent in the context of this problem?

c) Graph these two functions on the same graph. Plot and label their intersection. Use window Xmin=0, Xmax = 20, Ymin=1000, Ymax=2500.

d) When would the two investments return the same amount? How much would they return?

e) Which investment would you go with in the short term (less than 10 years)? Explain.

14. A certain bank compounds interest on the balances annually in all savings accounts. The function they use to compute the balance is $f(t)=P(1+r)^t$ where P is the principal and t is the time in years. If a savings account has a balance of \$5,000 and an annual interest rate of 6% use the function to find (show work and round to the nearest hundredth.):

a) The account balance after 7 years:

b) The amount of interest after 7 years:

c) Using the TI-84 calculator, after what time (t in years), will the account have \$6,400? Round to the hundredths place and sketch the situation below.

Window:

X min:_________________

X max:_________________

Y min:_________________

Y max:_________________

15. In 2010, the estimated population of Maricopa County was 3,817,117. By 2011, the population had grown to 3,880,244.

a) Assuming that the growth is linear, construct a linear equation that expresses the population, P, of Maricopa County t years since 2010.

b) Assuming that the growth is *exponential*, construct an exponential equation that expresses the population, P, of Maricopa County t years since 2010. Round the answer to three decimal places.

c) Use the equation found in part a) to predict the population of Maricopa County in 2015.

d) Use the equation found in part b) to predict the population of Maricopa County in 2015.

Name:________________________ **Date:**______________

Lesson 9 Assessment

1) Complete the following table. Use proper notation.

	$f(x) = 24(1.32)^x$	$f(x) = 3324(0.92)^x$	$f(x) = (1.04)^x$
Growth or Decay?			
Vertical Intercept (y-int)			
Horizontal Intercept (x-int)			
Domain			
Range			
Horizontal Asymptote (equation)			

2) Determine if each data set is linear or exponential, and write the formula for each.

x	$p(x)$
0	52
1	41
2	30
3	19

x	$g(x)$
0	128
1	64
2	32
3	16

x	$h(x)$
0	1000
1	1100
2	1210
3	1331

$p(x)=$ ____________

$g(x) =$ ______________

$h(x) =$ _____________

3) One 12-oz can of Dr. Pepper contains about 39.4 mg of caffeine. The function $A(x) = 39.4(0.8341)^x$ gives the amount of caffeine remaining in the body x hours after drinking a can of Dr. Pepper. Answer in complete sentences.

a) How much caffeine is in the body eight hours after drinking one can of Dr. Pepper? Show all of your work and write your answer in a complete sentence. Round your answer to two decimal places as needed.

b) Use the calculator to determine how long after drinking one can of Dr. Pepper will only 1 mg of caffeine remain in the body? Show all of your work, and write your answer in a complete sentence. Round your answer to two decimal places as needed.

c) Give the equation of the horizontal asymptote of $A(x)$. Explain the significance of the horizontal asymptote in this situation.

4) Solve algebraically and check graphically: $3^{-8x+5} = 81$

Lesson 10 – More Exponential Functions

Now that we have studied the basics of Exponential Functions, it is time to look at several specific concepts. In this lesson, we study Exponential Growth and Exponential Decay and look at ways to model and measure each. We also learn how to use our calculator to create an Exponential Model by using the Regression tool.

Lesson Topics:

Section 10.1: Writing Exponential Models

- Characteristics of Exponential Functions
- Growth/Decay Rates
- Writing Exponential Growth/Decay models

Section 10.2: Doubling Time and Halving Time

- Writing Exponential Growth models – Doubling Time
- Writing Exponential Decay models – Halving Time

Section 10.3: Exponential Regression

Mini-Lesson 10

Section 10.1 – Writing Exponential Models

Problem 1	**YOU TRY – Characteristics of Exponential Functions**

Given a function, $f(x) = ab^x$, respond to each of the following. Refer back to previous lessons as needed.

a) The variable x represents the ___________________ quantity.

b) $f(x)$ represents the __________________ quantity.

c) The DOMAIN of $f(x)$ is ___________________________________

d) The RANGE of $f(x)$ is ___________________________________

e) The INITIAL VALUE of $f(x)$ is ___________

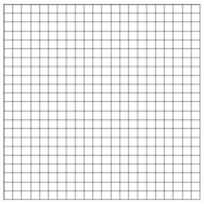
f) The VERTICAL INTERCEPT (Y-INT) of $f(x)$ is (_____ , _____)

g) The HORIZONTAL INTERCEPT (X-INT) of $f(x)$

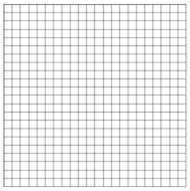
____

h) The equation of the HORIZONTAL ASYMPTOTE of $f(x)$ is ___________________

i) On the graphing grid below, draw an exponential GROWTH function. In this case, what can you say about the GROWTH FACTOR b? $b >$ ______



j) On the graphing grid below, draw an exponential DECAY function. In this case, what can you say about the DECAY FACTOR b? ______ $< b <$ ______



Growth and Decay RATES

An exponential function $f(x) = ab^x$ grows (or decays) at a constant *percent rate, r.*

r = growth/decay rate in decimal form

GROWTH FACTOR: $b = 1 + r$

GROWTH RATE: $r = b - 1$

DECAY FACTOR: $b = 1 - r$

DECAY RATE: $r = 1 - b$

Problem 2 MEDIA EXAMPLE – Writing Exponential Growth/Decay Models

Complete the following table.

Exponential Function $y = ab^t$	Growth or Decay?	Initial Value a	Growth/Decay Factor b	Growth/Decay Rate, r (as a decimal)	Growth/Decay Rate, r (as a %)
$y = 812(0.71)^t$					
$y = 64.5(1.32)^t$					
	Growth	8.24			0.5%
	Decay	150			20%

Problem 3	**WORKED EXAMPLE – Writing Exponential Growth/Decay Models**

Exponential Function $y = ab^t$	Growth or Decay?	Initial Value a	Growth/Decay Factor b	Growth/Decay Rate, r (as a decimal)	Growth/Decay Rate, r (as a %)
$f(t) = 72(1.03)^t$	Growth	72	1.03	0.03	3%
$f(t) = 44.1(0.92)^t$	Decay	44.1	0.92	0.08	8%
$f(t) = (0.54)^t$	Decay	1	0.54	0.46	46%
$f(t) = 2110(1.023)^t$	Growth	2110	1.023	0.023	2.3%
$f(t) = 520(0.85)^t$	Decay	520	0.85	0.15	15%
$f(t) = 3900(1.048)^t$	Growth	3900	1.048	0.048	4.8%

Problem 4	**YOU TRY – Writing Exponential Growth/Decay Models**

Complete the following table.

Exponential Function $y = ab^t$	Growth or Decay?	Initial Value a	Growth/Decay Factor b	Growth/Decay Rate, r (as a decimal)	Growth/Decay Rate, r (as a %)
$f(t) = 300(0.88)^t$					
$f(t) = 213(1.2)^t$					
	Growth	177			9.8%
	Decay	5.41			7%

Section 10.2 – Doubling Time and Halving Time

Problem 5	**MEDIA EXAMPLE – Writing Exponential Growth Models / Doubling Time**

In 2001, the population of a particular city was 22,395 with an identified growth rate of 6.2% per year. Assume that this growth rate is constant from year to year.

a) Write the EXPONENTIAL GROWTH MODEL for this situation.

b) What is the approximate population of the city in 2006? Round to the nearest person.

c) Estimate the number of years (to the nearest whole year) that it will take for the population to DOUBLE. In what actual year will this take place?

Problem 6	**WORKED EXAMPLE - Writing Exponential Growth Models/Doubling Time**

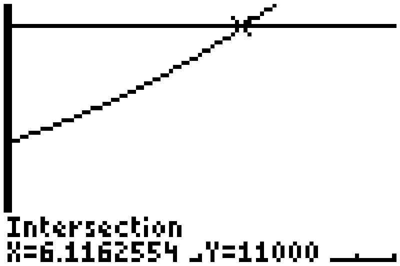
A city has a current population of 5500 people with a growth rate of 12% per year. Write the exponential model for this population and determine the time (to the nearest year) for the population to double.

First, determine the EXPONENTIAL MODEL using the information given in the problem.

- Given: Initial population = 5500
- Given: Growth rate of 12% per year
- Formula to use: $P(t) = ab^t$
- $a = 5500$ (initial population)
- To find b, convert 12% to a decimal (.12), Then, since the population grows, $b = 1 + .12 = 1.12$ (This value is also called the GROWTH FACTOR).
- Write the model: $P(t) = 5500(1.12)^t$

Second, determine the time for the population to double (DOUBLING TIME)

- Given: $P(t) = 5500(1.12)^t$, initial population = 5500
- Goal: Determine the time for the population to double. Another way to say this is, "find the value of t when the population is twice the initial population" (i.e. find t when $P(t) = 2(5500) = 11000$).
- Mathematically, we want to solve the equation: $5500(1.12)^t=11000$
- Use calculator entering Y1 = $5500(1.12)^t$ and Y2 = 11000. Use window X[0..10], Y[0..12000] then 2nd>Calc>5:Intersect to get $t = 6.12$. (See graph below). Round to get $t = 6$.



Intersection
X=6.1162554 Y=11000

Result: The population will double in about 6 years.

Steps to Write an Exponential Growth Model Given the Rate of Growth

- Determine initial value of the model (i.e. initial population, initial investment, initial salary, etc.). This is the value of the model at time $t = 0$ and the number will be your number for "a".
- Write the given rate as a decimal and ADD it to 1. This is your value for "b" (GROWTH FACTOR).
- Write the model using appropriate function notation (i.e. $P(t) = ab^t$, $V(t) = ab^t$, $S(t) = ab^t$, etc.)

Steps to Determine Doubling Time

- Start with an exponential growth model, i.e. $P(t) = ab^t$
- Set up the equation $ab^t = 2a$
- Solve by graphing and INTERSECTION method

Problem 7 YOU TRY – Writing Exponential Growth Models / Doubling Time

After graduating from college in 2010, Sara accepts a job that pays $52,000 per year. At the end of each year, she expects to receive a 3% raise.

a) Let t represent the number of years Sara works at her new job. Write the exponential growth function, *S(t),* that models her annual salary given the information above.

Initial Salary (a value): __________

Given growth rate as a decimal: ___________

Growth factor (b value): _______________

Write the model: $S(t)$ = InitialValue(GrowthFactor)^t = ____________________

b) If Sara's salary continues to increase at the rate of 3% each year, determine how much will she will make in 2015. Show your work clearly here.

c) How many years will she have to work before her salary will be double what it was in 2010 (assuming the same growth rate)? Be sure to set up and clearly identify the DOUBLING equation. Then, draw a sketch of the graph you obtain when using the INTERSECTION method to solve. Round to the nearest WHOLE year.

DOUBLING EQUATION: ____________________________________

DOUBLING TIME (Rounded to nearest whole year): _________________________

Problem 8	**MEDIA EXAMPLE – Writing Exponential Decay Models / Halving Time**

The 2000 U.S. Census reported the population of Tulsa, Oklahoma to be 382,872. Since the 2000 Census, Tulsa's population has been decreasing at approximately 2.6% per year.

a) Write an EXPONENTIAL DECAY MODEL, $P(t)$, that predicts the population of Tulsa, OK at any time t.

b) Use the function you wrote for $P(t)$ to predict the population of Tulsa, OK in 2013.

c) In how many years will the population of Tulsa decrease to 300,000 people (round to the nearest whole year)?

d) In how many years will the population of Tulsa decrease to HALF of the initial (2000) population? Round to the nearest whole year.

Problem 9 WORKED EXAMPLE – Writing Exponential Decay Models / Halving Time

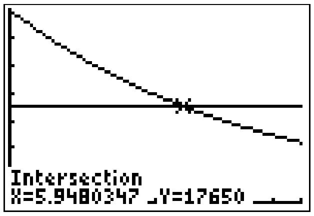
In 2012, Shannon purchased a new Ford Mustang GT convertible for $35,300. Since then, the value of the car has decreased at a rate of 11% each year.

First, determine the EXPONENTIAL MODEL using the information given in the problem.

- Given: Purchase price = $35,300
- Given: Decay rate of 11% per year
- Formula to use: $V(t) = ab^t$
- $a = 35{,}300$ (initial value)
- To find b, convert 11% to a decimal (0.11), Since the population decays, $b = 1 - .11 = 0.89$ (This value is also called the DECAY FACTOR).
- Write the model: $V(t) = 35300(0.89)^t$

Second, determine the time for the price to halve (HALF-LIFE or HALVING TIME)

- Given: $V(t) = 35300(0.89)^t$, initial price = $35,300
- Goal: Determine the time for the value of the car to halve. Another way to say this is, "find the value of t when the value is half the initial purchase price" (i.e. find t when $V(t) = 0.5(35{,}300) = 17{,}650$).
- Mathematically, we want to solve the equation: $35300(0.89)^t=17650$
- Use your calculator and enter $Y1 = 35300(0.89)^t$ and $Y2 = 17650$. Use window X[0..10], Y[0..35300] then 2nd>Calc>5:Intersect to get $t = 5.95$ (See graph below).



```
Intersection
X=5.9480347 Y=17650
```

- Result: The value of the car will be worth half the initial purchase price in about 6 years.

Steps to Write an Exponential Decay Model Given the Rate of Decay

- Determine initial value of the model (i.e. initial population, initial investment, initial salary, etc.). This is the value of the model at time $t = 0$ and the number will be your number for "a".
- Write the given rate as a decimal and SUBTRACT it from 1. This is "b"(DECAY FACTOR).
- Write the model using appropriate function notation (i.e. $P(t) = ab^t$, $V(t) = ab^t$, $S(t) = ab^t$, etc.)

Steps to Determine Halving Time (also called Half-Life)

- Start with an exponential growth model, i.e. $P(t) = ab^t$
- Set up the equation $ab^t = 0.5a$
- Solve by graphing and INTERSECTION method

Problem 10 YOU TRY – Writing Exponential Decay Models / Halving Time

In 1970, Buffalo, New York had a population of 462,768 people. Assume the population decreased by 1.4% each year from 1970 to 2030.

a) Let t represent the number of years since 1970 (i.e. your starting year is 1970 so t=0 in this year). Write the exponential decay function, $P(t)$, that models the annual population given the information above.

Initial Population (a value): __________ Given DECAY RATE as a decimal: ___________

Take 1 – the DECAY RATE decimal to identify your DECAY FACTOR (b value): _______________

Write the model: $P(t) = InitialValue(DecayFactor)^t =$ ____________________

b) If the population continues to decrease at the rate above, determine how many people lived in Buffalo in 1989. Show your work and round to the nearest person.

c) How many years will it take for the Buffalo population to decrease to half what it was in 1970 (assuming the same decay rate)? Be sure to set up and clearly identify the HALVING equation. Then, draw a sketch of the graph you obtain when using the INTERSECTION method to solve. Round to the nearest WHOLE year.

HALVING EQUATION: ___________________________

HALVING TIME (Rounded to nearest whole year): _________________________

Section 10.3 – Exponential Regression

As with LINEAR FUNCTIONS, we can work with a data table and, if appropriate, model that data using EXPONENTIAL REGRESSION. The steps are almost the same as those followed for LINEAR REGRESSION.

Problem 11	**MEDIA EXAMPLE–Exponential Regression**

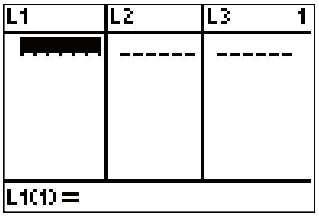
The table below shows the population, P, in a given state after t years.

t (years)	Population
5	5,234,456
10	4,892,345
15	4,012,345

Use the Exponential Regression feature of your calculator to generate a mathematical model for this situation. Round "a" to the nearest whole number and "b" to 3 decimals.

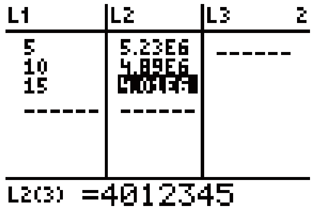
- Press STAT>EDIT>ENTER to show data entry area. The STAT button is on the second row, third column.

Data entry area should be blank to begin. To clear, go column by column. Scroll to column header using the arrow keys then press Clear>Enter. Use the arrow keys to move back and forth.



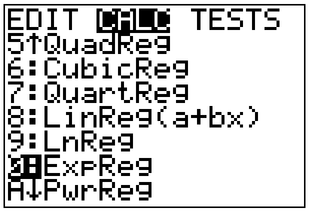
```
L1     |L2     |L3     1
------ |------ |------
L1(1) =
```

[Note: The numbers for L2 are displayed using scientific notation (the E notation) since they are too long for the column width. Scroll to each number and see its full representation in the bottom of the screen. See example highlighted at right.



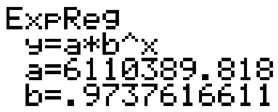
```
L1     |L2     |L3     2
5      |5.23E6 |------
10     |4.89E6 |
15     |4.01E6 |
------ |------ |
L2(3) =4012345
```

- Press STAT>CALC>0:ExpReg>ENTER>ENTER



Thus, your exponential function (with values rounded as the problem indicates) is $y = 6110390(0.974)^x$. Convert this to function notation with the appropriate variables to get

$$P(t) = 6110390(0.974)^t.$$



```
ExpReg
 y=a*b^x
 a=6110389.818
 b=.9737616611
```

Problem 12	**YOU TRY – Exponential Regression**

Determine the exponential regression equation that models the data below:

t	0	1	2	4	6	9	12	16
$P(t)$	125	75	50	32	22	16	10	5.7

When you write your final equation, round "a" to 1 decimal place and "b" to three decimal places.

a) Write exponential regression equation in the form $y = ab^x$: ______________________

Rewrite exponential regression equation in the form $P(t) = ab^t$: ____________________

b) Use your graphing calculator to generate a scatterplot of the data *and* the graph of the regression equation on the same screen. You must use an appropriate viewing window. In the space below, draw what you see on your calculator screen, and write down the viewing window you used.

Xmin=____________

Xmax=____________

Ymin=____________

Ymax=___________

c) What is the rate of decay (as a %) for this function? _________________________

d) Determine $P(20)$. Show your work, and write the corresponding ordered pair result. Round to two decimal places.

e) Using your equation from part a, determine t when $P(t) = 28$. Show your work. Write the corresponding ordered pair result. Round to two decimal places.

Problem 13	**YOU TRY – Exponential Regression**

The table below shows the value, V, of an investment (in thousands of dollars) after *n* years.

n	0	3	5	10	15	22
V(*n*)	4.63	5.92	6.88	10.23	15.21	26.39

a) Use your calculator to determine the exponential regression equation that models the set of data above. Round the "*a*" value to two decimals, and round the "*b*" value to three decimals. Use the indicated variables and proper function notation.

b) Based on the equation found in part a), at what percent rate is the value of this investment increasing each year?

c) Determine V(12) and interpret the meaning of V(12) by writing a complete sentence. Round your answer to two decimal places.

d) How long will it take for the value of this investment to reach $100,000? Round your answer to two decimal places. Write your answer in a complete sentence.

e) How long will it take for the value of the initial investment to double? Round your answer to two decimal places. Write your answer in a complete sentence.

Lesson 10 Practice Problems

Round to two decimal places unless stated otherwise.

Section 10.1: Writing Exponential Models

1. Complete the following table.

Growth Rate as a %	Growth Rate as a decimal	Growth Factor
13%	0.13	1.13
21%		
7%		
	0.20	
	0.05	
		1.25
		1.075
		2.03

2. Complete the following table.

Decay Rate as a %	Decay Rate as a decimal	Decay Factor
12%	0.12	0.88
23%		
3%		
	0.18	
	0.02	
		0.75
		0.98
		0.05

3. Write the exponential function for each of the following.

	Initial Value	Rate	Function
a)	1500	Growth Rate = 15%	
b)	75	Decay Rate = 15%	
c)	1250	Growth Rate = 7.5%	
d)	12	Growth Rate = 112%	
e)	1000	Decay Rate = 12%	
f)	56	Decay Rate = 5%	
g)	100	Decay Rate = 0.5%	
h)	57	Decay Rate = 6.2%	

4. For each exponential function, identify the Initial Value and the Growth/Decay Rate.

a) $f(x) = 1000(0.98)^x$

Initial Value =____________

Decay Rate = ____________

b) $g(x) = 3200(1.32)^x$

Initial Value =____________

Growth Rate = ____________

c) $p(t) = 50(0.75)^t$

Initial Value =____________

Decay Rate = ____________

d) $f(x) = 120(1.23)^x$

Initial Value =____________

Growth Rate = ____________

e) $A(r) = 1000(4.25)^r$

Initial Value =____________

Growth Rate = ____________

f) $g(x) = 1200(0.35)^x$

Initial Value =____________

Decay Rate = ____________

5. Complete the table below.

	Exponential Function	Growth or Decay?	Initial Value a	Growth/Decay Factor b	Growth/Decay Rate, r (as a decimal)	Growth/Decay Rate, r (as a %)
a)	$f(t)=45(0.92)^t$					
b)	$y=423(1.3)^t$					
c)		Growth	25			5.9%
d)		Decay	33.2			12.3%
e)			225	0.83		
f)			832	1.12		

6. When a new charter school opened in 2005, there were 300 students enrolled. **Using function notation**, write an exponential or linear equation to represent the number, N, of students attending this charter school *t* years after 2005, assuming that the student population

a) Decreases by 20 students per year.

b) Decreases by 2% per year.

c) Increases by 30 students per year.

d) Increases by 6% per year.

e) Decreases by 32 students per year.

f) Increases by 30% per year.

g) Remains constant (does not change).

h) Increases by 100% each year.

Section 10.2: Doubling Time and Halving Time

7. Determine the doubling or halving amount and the corresponding doubling or halving equation for the following functions.

	Function	Doubling or Halving Amount	Doubling or Halving Equation
a)	$f(t)=200(1.2)^t$		
b)	$f(x)=200(0.8)^x$		
c)	$g(t)=1500(1.5)^t$		
d)	$p(t)=3000(1.45)^t$		
e)	$g(x)=3000(0.99)^x$		
f)	$S(t)=25000(0.08)^t$		
g)	$h(t)=5.2(0.57)^t$		
h)	$A(t)=93.4(1.42)^t$		

8. Find the Doubling Time or Half Life for each of the following. Use the intersect feature on your graphing calculator. (You may use your doubling or halving equation from problem 7. Round your answer to two decimal places.)

a) $f(t) = 200(1.2)^t$

DOUBLING EQUATION: ___________________________

CORRESPONDING GRAPH:

Xmin = _______ Xmax = _______

Ymin = _______ Ymax = _______

DOUBLING TIME (Rounded to two decimal places): _________________________

b) $f(x) = 200(0.8)^x$

HALVING EQUATION: ___________________________

CORRESPONDING GRAPH:

Xmin = _______ Xmax = _______

Ymin = _______ Ymax = _______

HALVING TIME (Rounded to two decimal places): _________________________

c) $k(t)=1500(1.5)^t$

DOUBLING EQUATION: ____________________________

CORRESPONDING GRAPH:

Xmin = _______ Xmax = _______

Ymin = _______ Ymax = _______

DOUBLING TIME (Rounded to two decimal places): __________________________

d) $p(t)=3000(1.45)^t$

DOUBLING EQUATION: ____________________________

CORRESPONDING GRAPH:

Xmin = _______ Xmax = _______

Ymin = _______ Ymax = _______

DOUBLING TIME (Rounded to two decimal places): _______________________

e) $g(x)=3000(0.99)^x$

HALVING EQUATION: ___________________________

CORRESPONDING GRAPH:

Xmin = _______ Xmax = _______

Ymin = _______ Ymax = _______

HALVING TIME (Rounded to two decimal places): ______________________

f) $S(t)=25000(0.80)^t$

HALVING EQUATION: ___________________________

CORRESPONDING GRAPH:

Xmin = _______ Xmax = _______

Ymin = _______ Ymax = _______

HALVING TIME (Rounded to two decimal places): _________________________

g) $h(t) = 5.2(0.50)^t$

HALVING EQUATION: ____________________________

CORRESPONDING GRAPH:

Xmin = _______ Xmax = _______

Ymin = _______ Ymax = _______

HALVING TIME (Rounded to two decimal places): __________________________

h) $A(t) = 93.4(1.42)^t$

DOUBLING EQUATION: ____________________________

CORRESPONDING GRAPH:

Xmin = _______ Xmax = _______

Ymin = _______ Ymax = _______

DOUBLING TIME (Rounded to two decimal places): __________________________

i) $A(t)=5.24(2)^t$

DOUBLING EQUATION: ___________________________

CORRESPONDING GRAPH:

Xmin = _______ Xmax = _______

Ymin = _______ Ymax = _______

DOUBLING TIME (Rounded to two decimal places): _________________________

9. Amytown USA has a population of 323,000 in 1996. The growth rate is 8.4% per year. Show complete work for all problems.

   a) Find the exponential function for this scenario, $P(t) = ab^t$, where t is the number of years since 1996 and *P(t)* is the population *t* years after 1996.

   b) Determine the population of Amytown in 2013.

   c) Determine the year in which the population of Amytown will double.

10) Since 2003, the number of fish in Lake Beckett has been decreasing at a rate of 2.3% per year. In 2003, the population of fish was estimated to be 63.2 million. Show complete work for all problems.

    a) Find the exponential function for this scenario, $F(t) = ab^t$, where *t* is the number of years since 2003 and *F(t)* is the number of fish in millions *t years* after 2003.

    b) Determine the number of fish in Lake Beckett in 2020.

    c) Determine in what year the population of fish will be half the amount it was in 2003.

Section 10.3: Exponential Regression

11. Determine the exponential regression equation that models the data below:

t	0	1	2	4	6	9	12	16
$P(t)$	97	87	78	62	51	36	25	17

When you write your final equation, round "a" to one decimal place and "b" to three decimal places.

a) Write exponential regression equation in the form $y = ab^x$: ______________________

Rewrite exponential regression equation in the form $P(t) = ab^t$: ____________________

b) Use your graphing calculator to generate a scatterplot of the data *and* the graph of the regression equation on the same screen. You must use an appropriate viewing window. In the space below, draw what you see on your calculator screen, and write down the viewing window you used.

Xmin=____________

Xmax=____________

Ymin=____________

Ymax=___________

c) What is the rate of decay (as a %) for this function? _________________________

d) Using your regression model, determine $P(8)$.

e) Using your regression model, find t so that $P(t) = 40$. Show complete work.

12. The table below shows the value, V, of an investment (in thousands of dollars) after *n* years.

n	0	3	5	10	15	22
V(*n*)	3.74	4.58	5.24	7.46	10.21	17.01

a) Use your calculator to determine the exponential regression equation that models the set of data above. Round the "*a*" value to two decimals, and round the "*b*" value to three decimals. Use the indicated variables and proper function notation.

b) Based on the your regression model, what is the percent increase per year?

c) Find *V(8),* and interpret its meaning in a complete sentence. Round your answer to two decimal places.

d) How long will it take for the value of this investment to reach $50,000? Round your answer to two decimal places. Write your answer in a complete sentence.

e) How long will it take for the value of the investment to double? Round your answer to two decimal places. Write your answer in a complete sentence.

f) How long will it take for the value of the investment to triple? Round your answer to two decimal places. Write your answer in a complete sentence.

13. The following data represents the number of radioactive nuclei in a sample after t days.

t = time in days	0	1	4	6	9	12	17
N(t) = number of nuclei	2500	2287	1752	1451	1107	854	560

a) Use the exponential regression feature of your calculator to find the model of the form $N(t) = ab^t$. Round the "*a*" value to two decimals, and round the "*b*" value to three decimals. Use the indicated variables and proper function notation.

b) Using your model, find the number of nuclei after 5 days.

c) Using your model, find when there will be 1000 nuclei.

d) Use your model to find the number of nuclei after 9 days. How does this compare to the data value in the table?

e) Do the data values and regression values always match up? Why or why not?

Name:________________________ **Date:**_____________

Lesson 10 Assessment

1. Consider the functions shown below.

A. $f(x) = (1.023)^x$ B. $f(x) = 320(0.95)^x$ C. $f(x) = 400(1.12)^x$

D. $f(x) = 34.9(1.11)^x$ E. $f(x) = 172(0.99)^x$ F. $f(x) = 8452(0.67)^x$

a) Which functions are increasing? ________________________________

b) Which function is increasing at the fastest rate? ___________

What is the growth rate for this function? ___________

c) Which function is decreasing at the fastest rate? ___________

What is the decay rate for this function? ___________

2. Fred and Wilma purchase a home for $180,000. **Using function notation**, write a linear or exponential equation for the value, *V*, of the house *t* years after its purchase, assuming that the value:

a) Decreases by $1,500 per year. b**)** Decreases by 2% per year.

c) Increases by $3,100 per year. d) Increases by 6% per year

3. The following data set gives the value, V, of a car after *t* years.

Years since purchase	Value in Dollars
0	22,425
1	17,956
2	15,218
3	12,749
5	8,860
8	5,311

a) Determine an exponential regression equation of the form $V(t) = ab^t$ for this data set. Round the "*a*" value to the nearest whole number and the "*b*" value to three decimals.

b) Use the regression equation from part a) to predict the value of the car after 12 years. Round your answer to the nearest cent. Write your answer in a complete sentence.

c) Use the regression equation from part a) to predict how long until the car is worth half of its original value? Round your answer to the nearest hundredth. Write your answer in a complete sentence.

d) Use the regression equation from part a) to predict how long will it take for the car's value to reach $1000? Round your answer to the nearest hundredth. Write your answer in a complete sentence.

e) Based on the regression equation, at what percent rate is the car's value decreasing each year.

Lesson 11 – Course Review

In this lesson, we will review the topics and applications from Lessons 1-10. We will begin with a review of the different types of functions, and then apply each of them to a set of application problems. If you have forgotten how to work a particular type of problem, refer back to the corresponding Mini-Lesson for assistance.

Lesson Topics:

Section 11.1 Overview of Functions

- Linear, Exponential, Quadratic, Rational, and Radical Functions
- Identify basic characteristics and graph

Section 11.2 Solving Equations

- Graphically
- Algebraically

Section 11.3 Mixed Applications

Mini-Lesson 11

Section 11.1 – Overview of Functions

Problem 1	**YOU TRY – Linear Functions**

Complete the table. Write intercepts as ordered pairs. Use inequality notation for domain and range. Round to the nearest hundredth as needed. Write "N" if the answer does not exist.

	$f(x)=\frac{2}{3}x-6$	$g(x)=-4x$	$h(x)=103$
Behavior (Increasing, Decreasing, Horizontal, or Vertical)			
Slope			
Vertical Intercept (y-int)			
Horizontal Intercept (x-int)			
Domain			
Range			
Sketch the Graph on an appropriate viewing window. Label all intercepts and interesting features of the graph.			

Problem 2	**YOU TRY – Exponential Functions**

Complete the table. Write intercepts as ordered pairs. Use inequality notation for domain and range. Round to the nearest hundredth as needed. Write "N" if the answer does not exist.

	$f(x)=82(0.932)^x$	$g(x)=512(1.36)^x$
Growth or Decay?		
Growth / Decay Rate (as a %)		
Vertical Intercept (y-int)		
Horizontal Intercept (x-int)		
Asymptote (equation)		
Domain		
Range		
Sketch the Graph on an appropriate viewing window. Label all intercepts and interesting features of the graph.		

Problem 3	**YOU TRY – Quadratic Functions**

Complete the table. Write intercepts and the vertex as ordered pairs. Use inequality notation for domain and range. Round to the nearest hundredth as needed. Write “N” if the answer does not exist.

	$f(x)=x^2-8x+12$	$h(x)=-2x^2-31$	$g(x)=-(x-3)^2+4$
Opens Upward or Downward? Max or Min?			
Axis of Symmetry			
Vertex			
Vertical Intercept (y-int)			
Horizontal Intercept (x-int)(s)			
Domain			
Range			
Sketch the Graph on an appropriate viewing window. The vertex and intercepts must appear on the screen.			

Problem 4	**YOU TRY – Radical Functions**

Complete the table. Write intercepts as ordered pairs. Where applicable, give *both* the exact answer *and* the decimal approximation rounded to the nearest hundredth. Write "N" if the answer does not exist.

	$f(x)=\sqrt[3]{4x+9}$	$f(x)=\sqrt[4]{x-16}$	$f(x)=\sqrt{8-2x}$
Vertical Intercept (y-int)			
Horizontal Intercept (x-int)			
Domain			
Range			
Determine *f*(5)			
Determine *x* when *f*(*x*) = 5			
Sketch the Graph on an appropriate viewing window. Label all intercepts and interesting features of the graph.			

Problem 5	**YOU TRY –Rational Functions**

Complete the table. Write intercepts as ordered pairs. Use inequality notation for the domain. Round to the nearest hundredth as needed. Write "N" if the answer does not exist.

	$f(x)=\frac{4}{3x}$	$f(x)=\frac{4x-6}{5-x}$
Vertical Intercept (y-int)		
Horizontal Intercept (x-int)		
Domain		
Vertical Asymptote (equation)		
Horizontal Asymptote (equation)		
Determine *f*(8)		
Sketch the Graph on an appropriate viewing window. Label all intercepts and interesting features of the graph.		

Problem 6	**YOU TRY – Operations on Functions**

Complete the table. Simplify if possible.

Find	$f(x)=x-2$	$g(x)=x^2-5x+6$
$f(x)+g(x)$		
$f(x)-g(x)$		
$f(x)\cdot g(x)$		
$\frac{f(x)}{g(x)}$		

Problem 7	**YOU TRY – Operations on Functions**

Complete the table. Simplify if possible.

Find	$f(x)=\dfrac{4}{3x}$	$g(x)=\dfrac{4x-6}{5-x}$
$f(x)+g(x)$		
$f(x)-g(x)$		
$f(x)\cdot g(x)$		
$\dfrac{f(x)}{g(x)}$		

Section 11.2 – Solving Equations

Problem 8	**YOU TRY – Solving Equations by Graphing**

In each situation below, you will need to graph to find the solution to the equation using the INTERSECTION method. Fill in the missing information for each situation. Include a rough but accurate sketch of the graphs and intersection point. Mark and label the intersection. Round answers to two decimal places as needed.

a) Solve $3x^2 - 6x + 1 = 5$ Solutions: $x =$ _______________, $x =$ _______________

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

b) Solve $85(1.08)^x = 289$ Solution: $x =$ ______________

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

c) Solve $2+\sqrt{x+5}=9$ Solution: $x =$ ______________

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

Problem 9	**YOU TRY – Solving Equations Algebraically**

Solve the equations below ***algebraically*** showing all steps. Where applicable, give *both* the exact answer and the decimal approximation rounded to the nearest hundredth.

a) Solve $3x^2 - 6x + 1 = 5$

b) Solve $2^{x-1} = 32$

c) Solve $2 + \sqrt{x+5} = 9$

d) Solve $\frac{3}{x} + \frac{5}{6x} = 6$

e) Solve $\frac{2x-7}{x+1} = 3$

f) Solve $x^2 - 4x - 5 = 0$

Section 11.3 – Mixed Applications

Problem 10	**YOU TRY – Mixed Applications**

A toy rocket is shot straight up into the air. The function $H(t) = -16t^2 + 128t + 3$ gives the height (in feet) of a rocket after t seconds. Round answers to two decimal places as needed. All answers must include appropriate units of measure.

a) How long does it take for the rocket to reach its maximum height? Write your answer in a complete sentence.

b) What is the maximum height of the rocket? Write your answer in a complete sentence.

c) How long does it take for the rocket to hit the ground? Write your answer in a complete sentence.

d) Identify the Vertical Intercept (y-int). Write the vertical intercept as an ordered pair and interpret its meaning in a complete sentence.

e) Determine the practical domain of $H(t)$. Use inequality notation and include units.

f) Determine the practical range of $H(t)$. Use inequality notation and include units.

Problem 11	**YOU TRY – Mixed Applications**

The function $E(t) = 3861 - 77.2t$ gives the surface elevation (in feet above sea level) of Lake Powell t years after 1999. All answers must indicate the appropriate *year*.

a) Determine the surface elevation of Lake Powell in the year 2001. Show your work, and write your answer in a complete sentence. Round your answer to the nearest whole number.

b) Determine E(5), and write a sentence explaining the meaning of your answer. Round your answer to the nearest whole number.

c) Identify the Vertical Intercept (y-int) of this linear function. Write it as an ordered pair, then write a sentence explaining its meaning in this situation.

d) Identify the slope of this linear function and explain its meaning in this situation. Answer in a complete sentence and include all appropriate units.

e) This function accurately models the surface elevation of Lake Powell from 1999 to 2005. Determine the practical range of this linear function. Use proper inequality notation and **include units**. Round to the nearest whole number.

Problem 12	**YOU TRY – Mixed Applications**

One 12-oz can of Diet Pepsi contains about 36 mg of caffeine. The body metabolizes caffeine at a rate of about 14% per hour. Answer in complete sentences.

a) Write a formula for the amount, A, of caffeine remaining in the body x hours after drinking one can of Diet Pepsi. Your answer must be written in function notation.

b) Determine $A(3)$. Round your answer to two decimal places, and write a sentence explaining its meaning.

c) For what value of x is $A(x) = 3$? Round your answer to two decimal places, and write a sentence explaining its meaning.

d) How much caffeine is in the body one day after drinking one can of Diet Pepsi? Show all of your work and write your answer in a complete sentence. Round your answer to two decimal places as needed.

e) How long will it take the body to metabolize half of the caffeine from one can Diet Pepsi? Show all of your work and write your answer in a complete sentence. Round your answer to two decimal places as needed.

f) According to this model, how long will it take for *all* of the caffeine to leave the body?

Problem 13	**YOU TRY – Mixed Applications**

You and your family are heading out to San Diego on a road trip. From Phoenix, the trip is 355 miles according to Google. Answer the following questions based upon this situation. Round to the nearest tenth as needed.

a) Use the relationship, Distance = Rate times Time or $d = rT$, to write a rational function $T(r)$ that has the average rate of travel, r (in mph), as its input and the time of travel (in hours) as its output. The distance will be constant at 355 miles.

b) If you average 55 mph, how long will the trip take?

c) If the trip took 10 hours, what was your average rate of travel?

d) Determine the Vertical Intercept (y-int) of $T(r)$ and interpret its meaning. If the Vertical Intercept (y-int) does not exist, explain why (in the context of the story).

e) Determine the Horizontal Intercept (x-int) of $T(r)$ and interpret its meaning. If the Horizontal Intercept (x-int) does not exist, explain why (in the context of the story).

f) Give the *equation* of the vertical asymptote of $T(r)$, and write a sentence explaining its significance in this situation.

f) Give the *equation* of the horizontal asymptote of $T(r)$, and write a sentence explaining its significance in this situation.

Problem 14	**YOU TRY – Mixed Applications**

The table below shows the value, V, of an investment (in thousands of dollars) after *n* years.

n	0	3	5	10	15	22
V(*n*)	4.63	5.92	6.88	10.23	15.21	26.39

a) Use your calculator to determine the exponential regression equation that models the set of data above. Round the "*a*" value to two decimals, and round the "*b*" value to three decimals. Use the indicated variables and proper function notation.

b) Based on the equation found in part a, at what percent rate is the value of this investment increasing each year?

c) Determine V(12), and write your answer in a complete sentence. Round your answer to two decimal places.

d) Use the model to determine how long will it take for the value of this investment to reach \$100,000? Round your answer to two decimal places. Write your answer in a complete sentence.

e) How long will it take for the value of the investment to double? Round your answer to two decimal places. Write your answer in a complete sentence.

Problem 15	**YOU TRY – Mixed Applications**

In 2010, the estimated population of Maricopa County was 3,817,117. By 2011, the population had grown to 3,880,244.

a) Assuming that the growth is *linear*, construct a linear equation that expresses the population, P, of Maricopa County x years since 2010.

b) Assuming that the growth is *exponential*, construct an exponential equation that expresses the population, P, of Maricopa County x years since 2010. Round the answer to three decimal places.

c) Use the equation found in part a to predict the population of Maricopa County in 2015. Round to the nearest person.

d) Use the equation found in part b to predict the population of Maricopa County in 2015. Round to the nearest person.

Problem 16	**YOU TRY – Mixed Applications**

A resort hotel in Scottsdale, AZ charges \$1800 to rent a reception hall, plus \$58 per person for dinner and open bar. The reception hall can accommodate up to 200 people.

a) Write a function, *T,* to represent the total cost to rent the reception hall if *n* people attend the reception.

$T(n) =$ ______________________

b) During the summer months, the hotel offers a discount of 15% off the total bill, *T*. Write a function, *D,* to represent the discounted cost if the total bill was \$*T.*

$D(T)=$ ______________________

c) Using the information above, write a formula for $D(T(n))$ and complete the table below.

$D(T(n)) =$ ______________________________

n	0	50	100	150	200
D(*T*(*n*))					

d) What information does the function $D(T(n))$ provide in this situation? Be sure to identify the input and output quantities.

e) Interpret the meaning of the statement $D(T(100)) = 6460$. Include all appropriate units.

f) Determine the maximum number of people that can attend the reception for \$5,000 (after the discount is applied)?

Problem 17	**YOU TRY – Mixed Applications**

Marie can paint a house by herself in 12 hours. Abby can paint a house by herself in 16 hours. How long would it take them to paint the house if they worked together?

Name: ______________________ **Date:** ________________

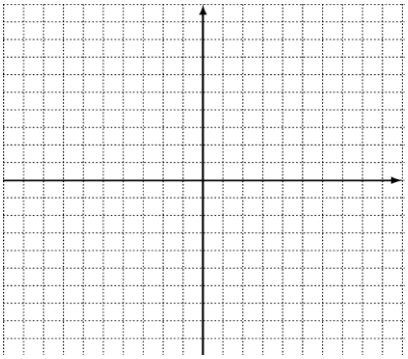
Lesson 11 Practice Problems

Round to two decimal places unless stated otherwise.

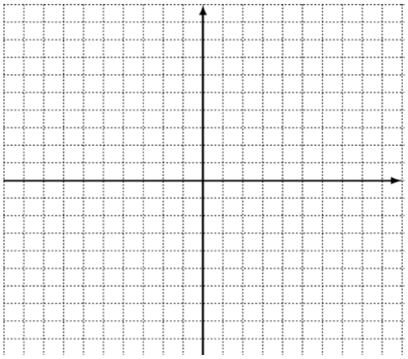
Functions

1. In the space below, draw a graph that represents a function, and a graph that does NOT represent a function.

Function



Not a Function



2. Are these functions? Circle yes or no.

Input	Output
1	5
2	5
3	5
4	5

Yes No

Input	Output
1	3
1	4
2	5
2	6

Yes No

Input	Output
-23	695
6	85
302	-80
12	0

Yes No

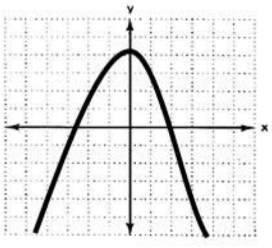
3. Are these functions? Circle yes or no.

a) {(2, -4), (6, -4), (0, 0), (5, 0)} Yes No

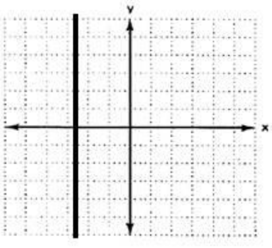
b) {(1, 1), (2, 2), (3, 3), (4, 4)} Yes No

c) {(1, -8), (5, 2), (1, 6), (7, -3)} Yes No

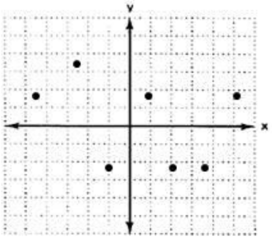
4. Are these functions? Circle yes or no.



Yes No



Yes No



Yes No

5. Answer true or false:

a) The sales tax is a function of the price of an item.

b) The numerical grade in this course is a function of the letter grade.

c) Cooking time for a turkey is a function of the weight of the bird.

d) The letter grade on a true/false quiz is a function of the number of questions answered correctly.

6. The function $r(x)$ is defined by the following table of values.

x	3	5	6	9	13
$r(x)$	-9	3	2	2	1

a) $r(9)$ = ____________ b) $r(3)$ = ______________

c) $r($ __________ $) = 1$ d) $r($ ____________ $) = 3$

e) The domain of $r(x)$ is { ____ }

f) The range of $r(x)$ is { ____ }

7. Consider the function $g = \{(2, 5),\ (0, 6),\ (5, 8),\ (-3, 7)\}$

a) $g(0)$ = ____________ b) $g(5)$ = ______________

c) $g($ __________ $) = 7$ d) $g($ ____________ $) = 5$

e) The domain of g is { ____ }

f) The range of g is { ____ }

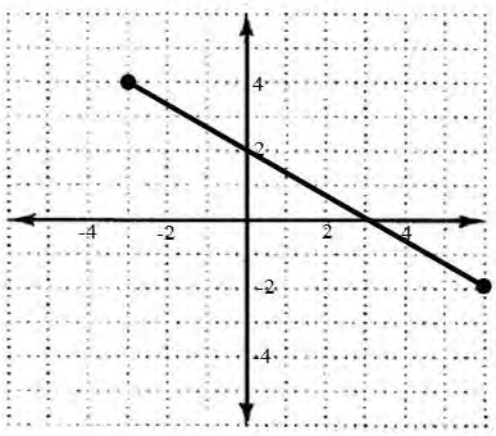
8. Given $f(4)=8,\ f(3)=11,\ f(0)=6$

a) The domain of *f* is { ____ }

b) The range of *f* is { ____ }

c) Write the function *f* as a set of ordered pairs.

9. The graph of $f(x)$ is given below.



a) Domain: ________________________

b) Range ________________________

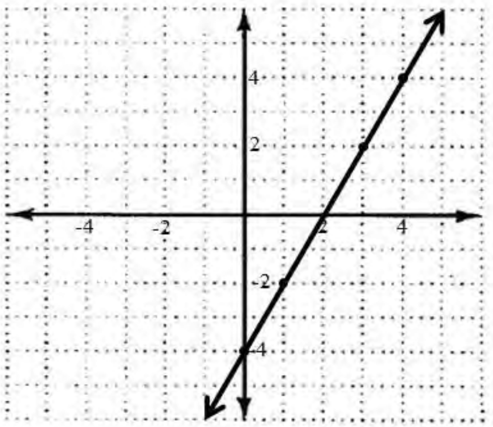
c) $f(-3) =$ _________

d) $f(0) =$ _________

e) $f(x) = 4$ when $x =$ __________

f) $f(x) = 0$ when $x =$ __________

10. The graph of $f(x)$ is given below.



a) Domain: ______________________

b) Range ______________________

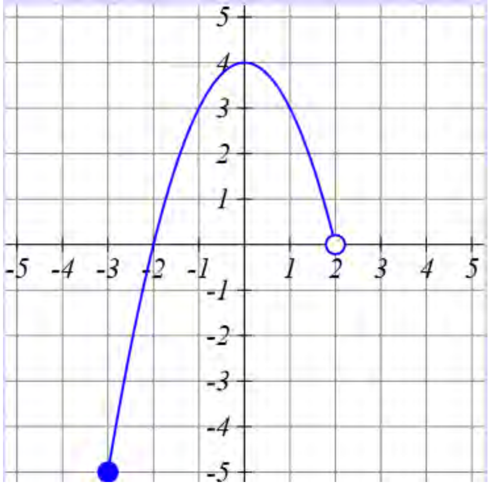
c) $f(3) =$ _________

d) $f(0) =$ _________

e) $f(x) = -2$ when $x =$ __________

f) $f(x) = 0$ when $x =$ __________

11. The graph of $f(x)$ is given below.



a) Domain: ______________________

b) Range ______________________

c) $f(-1) =$ _________

d) $f(0) =$ _________

e) $f(x) = -5$ when $x =$ __________

12. Let $W(p) = p^2 - 9p + 20$. Show all steps. Write each answer in function notation *and* as an ordered pair.

a) Determine W(-10) .

b) For what value(s) of p is W(p) = 0?

13. Let $h(x) = x^2 - 7x + 9$. Show all steps. Write each answer in function notation *and* as an ordered pair.

a) Determine $h(5)$.

b) Determine $h(81)$.

14. Let $p(x) = \frac{40}{2x}$. Show all steps. Write each answer in function notation *and* as an ordered pair.

a) Determine $p(5)$.

b) For what value of x is $p(x) = \frac{1}{4}$?

c) Determine the domain of $p(x)$.

15. The functions A and B are defined by the following tables

x	–3	–2	0	1	4	5	8	10	12
$A(x)$	8	6	3	2	5	8	11	15	20

x	0	2	3	4	5	8	9	11	15
$B(x)$	1	3	5	10	4	2	0	–2	–5

Determine the values for each of the following.

a) $B(3)$=_________

b) $A(8)$=_________

c) $A(0)+B(0)$=_______

d) $A(8) - B(8)$=_________

e) $A(4) \cdot B(4)$=_________

f) $\frac{A(5)}{B(5)}$ =________

g) $A(B(0))$=_________

h) $B(A(10))$=_________

i) $B(B(3))$=________

16. Let $p(x) = x^2 + 2x + 3$ and $r(x) = x - 5$. Determine each of the following. Show all work. Simplify your answers.

a) $p(x) - r(x)$=

b) $p(0) \cdot r(0)$=

c) $p(-2) + r(-2)=$

d) $r(7) - p(7)=$

e) $p(r(x))=$

f) $r(p(7))=$

Linear Functions

17. Darby signs a 48-month lease agreement for a new Chevrolet Camaro 2LT convertible. The function $T(n) = 3491.88 + 580.85n$ gives the total amount paid n months after signing.

a) Using complete sentences, interpret $T(12)=10462.08$ in the context of the story.

b) Determine $T(24)$ and write a sentence explaining the meaning of your answer in this situation.

c) Determine the value of n if $T(n) = 30{,}000$. Write a sentence explaining the meaning of your answer in this situation.

d) Identify the slope of $T(n)$ and interpret its meaning in a complete sentence.

e) Identify the Vertical Intercept (y-int) of $T(n)$. Write it as an ordered pair and interpret its meaning in a complete sentence.

f) Determine the practical domain of $T(n)$. Use inequality notation. Include units.

g) Determine the practical range of $T(n)$. Use inequality notation. Include units.

18. A candy company has a machine that produces candy canes. The table below is a partial list of the relationship between the number of minutes the machine is operating and the number of candy canes produced by the machine during that time period.

Minutes t	3	5	8	12	15
Candy Canes $C(t)$	12	20	32	48	60

a) Include units. $C(12)$ = ________________________________

b) In a complete sentence and including all appropriate units, explain the meaning of your answer in part a).

c) Determine the average rate of change of C(t) from 5 minutes to 8 minutes. Interpret your answer in a complete sentence.

d) Is $C(t)$ a linear function? If yes, identify the slope explain, in complete sentence, what slope means in the context of the problem.

e) Find the vertical intercept and explain, in complete sentence, what vertical intercept means in the context of the problem.

f) Write the equation that models this scenario in $C(t) = mt + b$ form .

19. The following table shows the distance of rocket from Earth in 100,000's of miles as it travels towards Mars.

Number of Days	1	3	7	12	18	27	37	44
Distance from Earth (100,000's of miles)	5.1	15.9	35.2	61.1	89.7	137.2	183.5	223.0

a) Let t = the number of days since the rocket was launched and $D(t)$ = the distance from Earth. Use your calculator to determine the linear regression equation for the data (round all values to two decimal points).

b) Use your regression equation to estimate the rocket's distance from Earth 23 days after launch.

c) Use your regression equation to estimate when the rocket will reach Mars if Mars' distance from Earth is approximately 127 million miles.

d) Determine $D(20)$ and write a sentence explaining the meaning of your answer.

e) Use your regression equation to estimate the velocity of the rocket. Make sure to include units in your answer.

20. Bill's car breaks down and he calls a tow company. The company's charges can be found by using the linear function, $T(x) = 5.50x + 24.50$ where $T(x)$ is the cost in dollars and x is the number of miles the car is towed.

a) Identify Vertical Intercept (y-int) of this function. Write it as an ordered pair and explain its meaning in the context of this problem.

b) Identify the slope of this function. Explain its meaning in the context of this problem.

21. Find the equation of the line passing through (–3,3) and (5,7). Leave your answer in $y = mx + b$ form.

22. Find the equation of the horizontal line passing through (–3, 2).

23. Find the equation of the vertical line passing through (–3, 2).

24. The function $d(t) = -63.24t + 874.9$ can be used to determine Donna's distance, in miles, from Phoenix as she is traveling home from her summer vacation in Idaho, after t hours of driving.

a) Evaluate *d(10)* and interpret its meaning in the context of the problem.

b) Find *t* so that *d(t) = 0* and interpret it's meaning in the context of the problem. Round to the nearest hour.

c) Find the slope of the function and interpret its meaning in the context of the problem.

d) Find the Vertical Intercept (y-int) of the function and interpret its meaning in the context of the problem.

e) Suppose Donna wants to make the trip home in 10 hours. How much faster would she need to travel? Explain.

Exponential Functions

25. Complete the following table. Use proper notation. Write "N" if the answer does not exist.

	$f(x)=24(1.32)^x$	$f(x)=3324(0.92)^x$	$f(x)=(1.04)^x$
Growth or Decay?			
Growth or Decay Rate (as a percent)			
Vertical Intercept (y-int)			
Horizontal Intercept (x-int)			
Domain			
Range			
Horizontal Asymptote (equation)			

26. Determine if each data set is linear or exponential and write the formula for each. Show complete work.

a)

x	−2	−1	0	1	2	3	4
$f(x)$	.04	.2	1	5	25	125	625

b)

x	−2	−1	0	1	2	3	4
$f(x)$	−1.375	−.5	.375	1.25	2.125	3	3.875

c)

x	−2	−1	0	1	2	3	4
$f(x)$	−3	−5.5	−8	−10.5	−13	−15.5	−18

d)

x	0	1	2	3	4	5	6
$f(x)$	2	4	8	16	32	64	128

27. Fred and Wilma purchase a home for \$150,000. **Using function notation**, write a formula for the value, *V*, of the house *t* years after its purchase, assuming that the value

a) Decreases by \$1,000 per year.

b) Decreases by 3% per year.

c) Increases by \$3,000 per year.

d) Increases by 5% per year.

28. The function $f(x) = 300(1.15)^x$ gives the population of a city (in thousands) *x* years since 2000.

a) Identify the Vertical Intercept (y-int). Write it as an ordered pair and interpret its meaning in this situation.

b) Is the population of this city increasing or decreasing? At what rate? Write your answers in complete sentences and include all appropriate units.

c) Determine *f*(10) and write a sentence explaining the meaning of your answer.

d) How long will it take the population of this city to reach one million? Show all work and write your answer in a complete sentence.

e) How long will it take the population to double? Show all work and write your answer in a complete sentence.

29. You purchased a vehicle for $30,000. Assuming the value of the car decreases at 8% per year, write an equation that represents the value, $V(t)$, of the car t years from now. How many years does it take for the value to decay to $20,000? Round to the nearest hundredth.

30. Determine an exponential regression function, P(t) to represent the data below. Let t=0 in the year 1930. Round "a" to the nearest whole number and "b" to 4 places. In what year will the population reach 5000 (round to the nearest whole year)?

Year	1930	1940	1980	1990	2000
Population	25908	25622	19057	17533	16328

31. In each situation a), b) below, you will need to graph to find the solution to the equation using the INTERSECTION method. Fill in the missing information for each situation. Include a rough but accurate sketch of the graphs and intersection point. Mark and label the intersection. Round answers to two decimal places. For c), d) solve algebraically.

a) Solve $54(1.05)^x = 250$ Solution: $x =$ ______________

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

b) Solve $2340(0.82)^x = 1250$ Solution: $x =$ ______________

Xmin:__________

Xmax:__________

Ymin:__________

Ymax:__________

c) Solve and check: $3^{x-5} = 81$

d) Solve: $5^{-3x+3} = 125$

Quadratic Functions

32. Fill out the following table. Intercepts must be written as ordered pairs. Always use proper notation. Round to two decimal places where necessary.

	$f(x)=x^2-5x+4$	$g(x)=16-x^2$	$y = -x^2 - 2x + 5$	$p(x)=2(x-4)^2+2$
Opens Upward or Downward? Max or Min				
Vertical Intercept (y-int)				
Horizontal Intercept (x-int)(s)				
Vertex				
Domain				
Range				
Axis of Symmetry (Equation)				

33. Factor each of the following. Write your answer in completely factored form.

a) $3x^2 - 9x$

b) $x^2 - 4x + 3$

c) $x^2 + x - 30$

d) $x^2 - 9$

e) $2x^2 + 7x + 6$

f) $x^2 - 144$

g) $3x^2 + 2x - 5$

h) $-2x^2 + 200$

i) $4x^2 - x - 5$

j) $5y^2 + 11y + 2$

34. Solve $x^2 + 18x - 68 = 20$ using the methods indicated below. Show all work.

a) Solve by graphing. Sketch the graph on a good viewing window (the vertex, intercepts and intersection points must appear on the screen). Mark and label the solutions on your graph.

Xmin:_________ Xmax: _________

Ymin: _________ Ymax: _________

Solution(s): ___________________________

b) Solve by factoring.

c) Use the quadratic formula to solve.

35. Solve the following equations *algebraically* (Factoring or Quadratic Formula or Square Root Property). You must show all algebraic steps for full credit. Where applicable, give both the exact answers and the decimal approximations rounded to three decimal places. Use your calculator to *check* your answers. Sketch the graph on a good viewing window (the vertex, Vertical Intercept (y-int), and any Horizontal Intercept (x-int)s should appear on the screen). **Mark and label** any real solutions on the graph.

a) $2x^2 - 8x + 10 = 4$

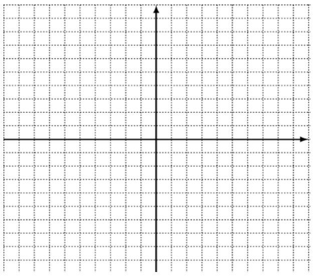
b) $2x^2 = -6x$

c) $x^2 + 4 = 4x$

d) $x^2 - 3x = 10$

e) $2(x-4)^2 = 6$

36. Given a quadratic equation in the form $f(x) = ax^2 + bx + c$, draw the graph of a parabola where $a > 0$ and $c < 0$.



37. Suppose $h(t) = -16t^2 + 40t + 80$ represents the height of a ball (measured in feet above the ground) thrown from a roof as a function of time (in seconds).

a) Find the value(s) of t such that $h(t) = 24$. Interpret your results in the context of this problem.

b) Write the equation you would solve to determine when the ball will hit the ground. Solve this equation to an accuracy of two decimal places. Show your work.

c) Determine the maximum height of the ball. Explain how you found this.

d) Determine the practical domain and range of $h(t)$

Radical Functions

38. Complete the table. Write intercepts as ordered pairs. Use inequality notation for domain and range. Round to the nearest hundredth as needed. Write "N" is the answer does not exist.

	$f(x)=\sqrt[5]{3x}$	$f(x)=\sqrt{x+9}$	$f(x)=\sqrt[3]{12-x}$
Vertical Intercept (y-int)			
Horizontal Intercept (x-int)			
Domain			
Determine *f*(5)			
Determine *x* when *f*(*x*) = 3			
Sketch the Graph on an appropriate viewing window. Label all intercepts and interesting features of the graph.			

39. Solve each of the equations algebraically. Show all of your work. Round any decimal results to three places. Write your answers in exact form.

a) $6+\sqrt[3]{7-3x}=16$

b) $\sqrt{8x-7}=x$

c) $4\sqrt{x-6}=12$

d) $\sqrt[4]{2x+8}+5=0$

e) $\sqrt{2x+10}+5=x+6$

f) $5-\sqrt[3]{5x}=11$

Rational Functions

40.

Find and Simplify if possible	$f(x)=\dfrac{6}{3x}$	$g(x)=\dfrac{2x+12}{4-x}$
$f(x)+g(x)$		
$f(x)-g(x)$		
$f(x)\cdot g(x)$		
$\dfrac{f(x)}{g(x)}$		

41. Complete the table. Write intercepts as ordered pairs. Round to the nearest hundredth as needed. Write “N” is the answer does not exist.

	$f(x)=\frac{6}{3x}$	$f(x)=\frac{2x+12}{4-x}$
Vertical Intercept (y-int)		
Horizontal Intercept (x-int)		
Domain		
Vertical Asymptote (equation)		
Horizontal Asymptote (equation)		
Determine $f(5)$		
Determine x when $f(x) = 5$		

42. Solve the following equations *algebraically*. You must show all algebraic steps for full credit. Where applicable, give both the exact answers and the decimal approximations rounded to three decimal places. Use your calculator to *check* your answers. Sketch the graph on a good viewing window. **Mark and label** the solution(s) on the graph.

a) $\frac{5}{9}=\frac{8}{x+1}$

b) $4=7+\frac{2}{x-1}$

c) $\frac{2}{x}+\frac{3}{x^2}=1$

Name ____________________________ **Date**_______________

Lesson 11 Assessment

1. Solve and check: $\sqrt{2x+8}=6$

2. Solve by using the quadratic formula and by factoring: $2x^2+9x=-7$

3. Bill's car breaks down and he calls a tow company. The company's charges can be found by using the linear function, $T(x)=6x+32$ where $T(x)$ is the cost in dollars and x is the number of miles the car is towed.

a) Identify Vertical Intercept (y-int) of this function. Write it as an ordered pair and explain its meaning in this context by writing a complete sentence.

b) Identify the slope of this function. Explain its meaning in the context by writing a complete sentence.

4. A toy rocket is shot straight up into the air. The function $H(t) = -16t^2 + 64t + 7$ gives the height (in feet) of a rocket after *t* seconds. Round answers to two decimal places as needed. All answers must include appropriate units of measure.

a) How long does it take for the rocket to reach its maximum height? Write your answer in a complete sentence.

b) What is the maximum height of the rocket? Write your answer in a complete sentence.

c) How long does it take for the rocket to hit the ground? Write your answer in a complete sentence.

d) $H(2) = 71$. Write a complete sentence to explain what this means in context of the problem.

e) Identify the Vertical Intercept (y-int). Write it as an ordered pair and interpret its meaning in a complete sentence.

f) Determine the practical domain of $H(t)$. Use inequality notation and include units.

g) Determine the practical range of $H(t)$. Use inequality notation and include units.

5. Solve the following equations *algebraically*. You must show all algebraic steps for full credit. Where applicable, give both the exact answers and the decimal approximations rounded to three decimal places. Use your calculator to *check* your answers. Sketch the graph on a good viewing window. **Mark and label** the solution(s) on the graph.

$$5=\frac{8}{x+1}-7$$

6. Simplify: $h\left(x\right)=\frac{x^2-3x-4}{x+1}$

7. Find the domain of the function: $g\left(x\right)=\frac{x^2-3x-4}{2x-5}$

8. Let $f(x)=\frac{2}{x+2}$ and $g(x)=\frac{3}{2x-5}$, find and simplify $f(x)-g(x)$.

9. Simplify and write the answer with a single positive rational exponent: $x^{1/4} \cdot x^{2/3}$

10. A) Find the vertex, equation of the axis of symmetry, horizontal intercept(s) (EXACT and APPROXIMATE form), vertical intercept, domain, and range of the function:
$f(x) = -2(x-3)^2 + 4$

Vertex:____________

Equation of the Axis of Symmetry:_____________

Domain:____________

Since a=_____ Circle One: Opens Up or Opens Down

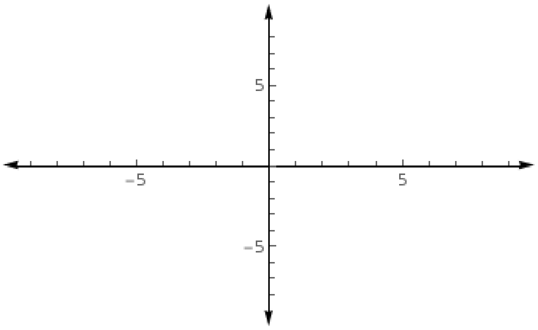
Circle One: Maximum or Minimum

Range:_____________

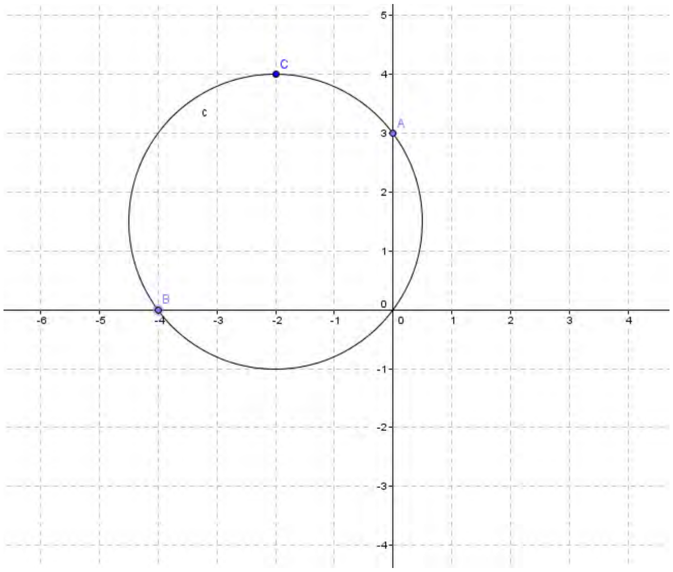
Horizontal Intercept(s) (x-int(s)) (Square Root property):

Vertical Intercept(y-int) (algebraically):

B) Draw a graph of the function labeling the Vertex, Axis of Symmetry, and two other points.



11. Given the graph below identify:



a) Horizontal Intercept(s):

b)Vertical Intercept(s):

c) Is the graph of a function? Explain your answer by writing a complete sentence.

d) Domain:

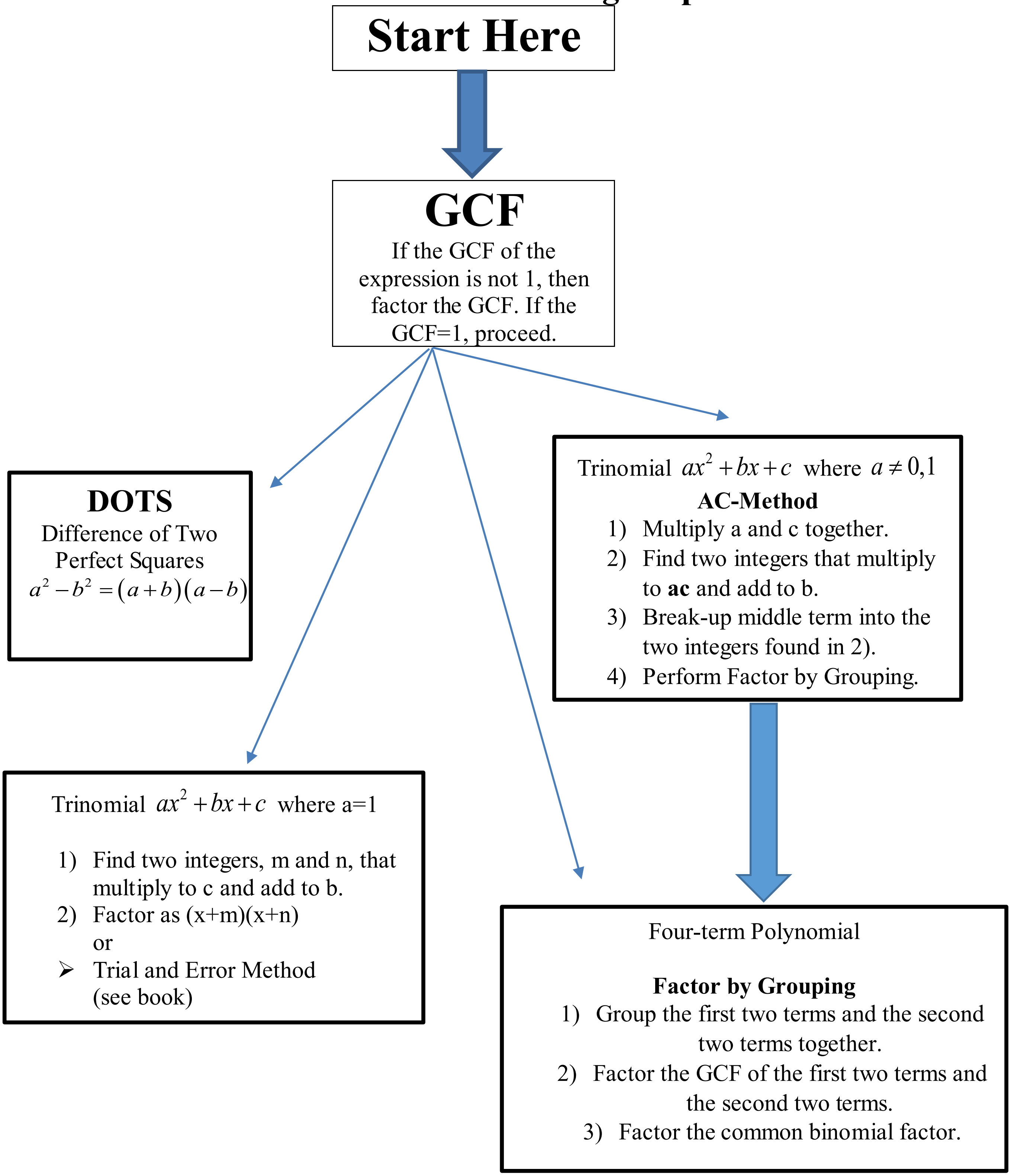
e) Range:

12.Collen can paint a room in 12 hours. Rebecca can paint the same room in 20 hours. How long does it take for both Collen and Rebecca to paint the room if they are working together? Write an equation that using the variable t to represent the time it takes to paint the room together that can be used to solve the problem.

13. Police use the formula $v = \sqrt{20L}$ to estimate the speed of a car, v, in miles per hour, based on the length L, in feet, of its skid marks when suddenly braking on a dry, asphalt road. At the scene of an accident, a police officer measures a car's skid marks to be 184 feet long. Approximately how fast was the car driving? Round the answer to the nearest tenth of a unit.

Appendix

Section 6.3:Factoring Map



Start Here

GCF

If the GCF of the expression is not 1, then factor the GCF. If the GCF=1, proceed.

DOTS

Difference of Two Perfect Squares

$a^2 - b^2 = (a+b)(a-b)$

Trinomial $ax^2 + bx + c$ where $a \neq 0,1$

AC-Method

1) Multiply a and c together.
2) Find two integers that multiply to **ac** and add to b.
3) Break-up middle term into the two integers found in 2).
4) Perform Factor by Grouping.

Trinomial $ax^2 + bx + c$ where a=1

1) Find two integers, m and n, that multiply to c and add to b.
2) Factor as (x+m)(x+n)

or

- Trial and Error Method (see book)

Four-term Polynomial

Factor by Grouping

1) Group the first two terms and the second two terms together.
2) Factor the GCF of the first two terms and the second two terms.
3) Factor the common binomial factor.

Section 6.5: Simplifying Square Roots

As a review, you may complete the following questions in preparation for section 6.5.
For each square root below:

a) Simplify, if possible.
b) Find the decimal approximation rounded to the nearest hundredth, if necessary.

1) $\sqrt{16}$

2) $\sqrt{64}$

3) $\sqrt{40}$

4) $\sqrt{60}$

5) $\sqrt{19}$

6) $\sqrt{78}$

7) $\sqrt{20}$

8) $\sqrt{144}$

9) $\sqrt{120}$

10) $\sqrt{99}$

11) $\sqrt{56}$

12) $\sqrt{76}$

13) $\sqrt{90}$

14) $\sqrt{68}$

15) $\sqrt{18}$

Overview of Functions

Functions	Parent Functions
Polynomial Functions Linear Function: Slope-Intercept form - $f(x) = mx + b$ Quadratic Function: General Form - $f(x) = ax^2 + bx + c$ Standard (Vertex) Form - $f(x) = a(x-h)^2 + k$	**Polynomial Functions** Linear Function: $f(x) = x$ Quadratic Function: $f(x) = x^2$
Other Functions Radical Function: $f(x) = \sqrt[q]{g(x)}$ Rational Function: $h(x) = \dfrac{f(x)}{g(x)}$, $g(x) \neq 0$ Exponential Function: $g(x) = ab^x$	**Other Functions** Radical Function: $f(x) = \sqrt[q]{x}$ Rational Function: $h(x) = \dfrac{1}{x}$, $x \neq 0$ Exponential Function: $g(x) = b^x$

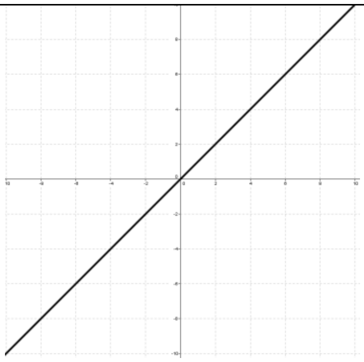
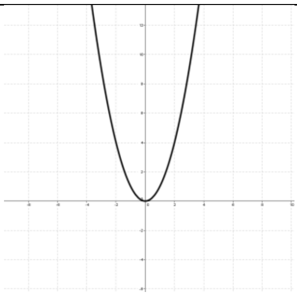
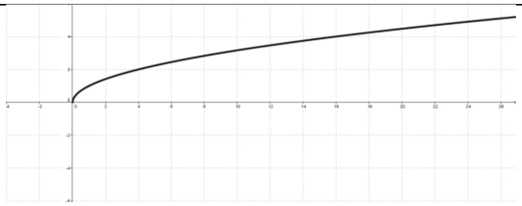
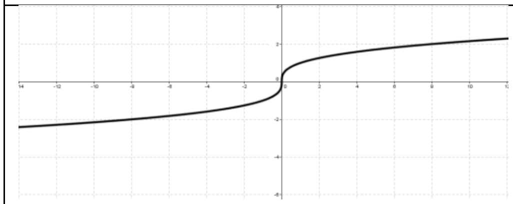
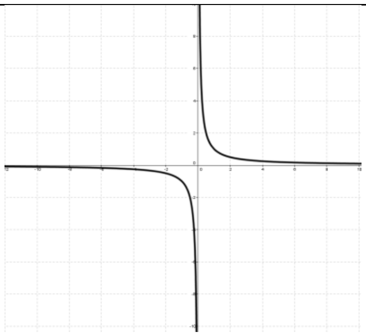
Formulas

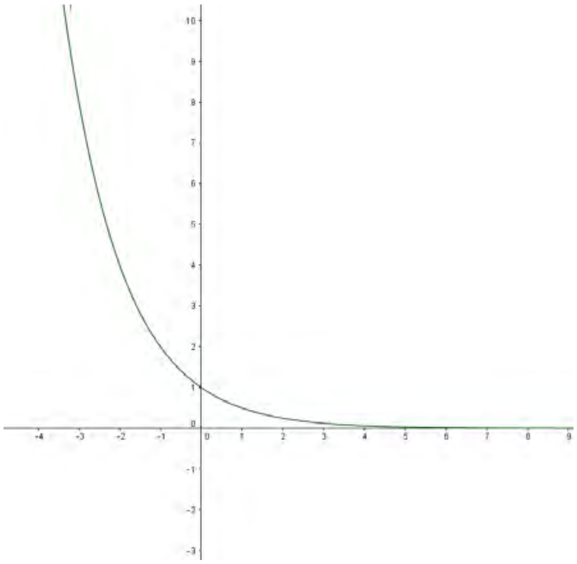
Average Rate of Change

$$A.R.O.C = slope = m = \frac{Rise/Fall}{Run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Quadratic Function (General Form) $f(x) = ax^2 + bx + c$ Axis of Symmetry: $x = -\dfrac{b}{2a}$ Vertex: $\left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$ Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Function (Standard (Vertex) Form) $f(x) = a(x-h)^2 + k$ Vertex: (h, k) Axis of Symmetry: $x = h$

Key Properties of Parent Functions

Function	Domain	Range	Asymptote	Graph
Linear $f(x)=x$	$(-\infty,\infty)$	$(-\infty,\infty)$	DNE	
Quadratic $f(x)=x^2$	$(-\infty,\infty)$	$[0,\infty)$	DNE	
Radical $f(x)=\sqrt[q]{x}$	If q is a positive even integer: $[0,\infty)$	If q is a positive even integer: $[0,\infty)$	DNE	
	If q is a positive odd integer: $(-\infty,\infty)$	If q is a positive odd integer: $(-\infty,\infty)$		
Rational $h(x)=\frac{1}{x}$	The set of all real numbers except x=0.	The set of all real numbers except y=0.	Vertical: x=0 Horizontal: y=0	

Function	Domain	Range	Asymptote	Graph
Exponential $g(x)=b^x$	$(-\infty,\infty)$	$(0,\infty)$	Horizontal: y=0	If $0<b<1$ (Decay, Decreasing):  If $b>1$ (Growth, Increasing) :